

Course Title: Analysis of Indeterminate Structures

[As per Choice Based Credit System (CBCS) scheme]

SEMESTER:V

Subject Code	15CV52	IA Marks	20
Number of Lecture Hours/Week	04	Exam Marks	80
Total Number of Lecture Hours	50	Exam Hours	03
CREDITS – 04		Total Marks-100	
Course objectives: This course will enable students to			
<ol style="list-style-type: none"> 1. Ability to apply knowledge of mathematics and engineering in calculating slope, deflection, bending moment and shear force using slope deflection, moment distribution method and Kani's method. 2. Ability to identify, formulate and solve problems in structural analysis. 3. Ability to analyze structural system and interpret data. 4. Ability to use the techniques, such as stiffness and flexibility methods to solve engineering problems 5. Ability to communicate effectively in design of structural elements 			
Modules	Teaching Hours	Revised Bloom's Taxonomy (RBT) Level	
Module -1			
Slope Deflection Method: Introduction, sign convention, development of slope deflection equation, analysis of continuous beams including settlements, Analysis of orthogonal rigid plane frames including sway frames with kinematic indeterminacy ≤ 3	10 hours	L ₂ , L ₄ ,L ₅	
Module -2			
Moment Distribution Method: Introduction, Definition of terms, Development of method, Analysis of continuous beams with support yielding, Analysis of orthogonal rigid plane frames including sway frames with kinematic indeterminacy ≤ 3	08 Hours	L ₂ , L ₄ ,L ₅	
Module -3			
Kani's Method: Introduction, Concept, Relationships between bending moment and deformations, Analysis of continuous beams with and without settlements, Analysis of frames with and without sway	08 Hours	L ₂ , L ₄ ,L ₅	
Module -4			
Matrix Method of Analysis (Flexibility Method) : Introduction, Axes and coordinates, Flexibility matrix, Analysis of continuous beams and plane trusses using system approach, Analysis of simple orthogonal rigid frames using system approach with static indeterminacy ≤ 3	12 Hours	L ₂ , L ₄ ,L ₅	
Module -5			
Matrix Method of Analysis (Stiffness Method): Introduction, Stiffness matrix, Analysis of continuous beams and plane trusses using system approach, Analysis of simple orthogonal rigid frames using system approach with kinematic indeterminacy ≤ 3	12 Hours	L ₂ , L ₄ ,L ₅	

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B G Nagara - 571448**

Course outcomes: After studying this course, students will be able to:

1. Determine the moment in indeterminate beams and frames having variable moment of inertia and subsidence using slope deflection method
2. Determine the moment in indeterminate beams and frames of no sway and sway using moment distribution method.
3. Construct the bending moment diagram for beams and frames by Kani's method.
4. Construct the bending moment diagram for beams and frames using flexibility method
5. Analyze the beams and indeterminate frames by system stiffness method.

Program Objectives:

- Engineering knowledge
- Problem analysis
- Interpretation of data

Question paper pattern:

- The question paper will have 5 modules comprising of ten questions. Each full question carrying 16 marks
- There will be two full questions (with a maximum of three subdivisions, if necessary) from each module.
- Each full question shall cover the topics as a module
- The students shall answer five full questions, selecting one full question from each module. If more than one question is answered in modules, best answer will be considered for the award of marks limiting one full question answer in each module.

Text Books:

1. Hibbeler R C, "Structural Analysis", Pearson Publication
2. L S Negi and R S Jangid, "Structural Analysis", Tata McGraw-Hill Publishing Company Ltd.
3. D S Prakash Rao, "Structural Analysis: A Unified Approach", Universities Press
4. K.U. Muthu, H.Narendra etal, "Indeterminate Structural Analysis", IK International Publishing Pvt. Ltd.

Reference Books:

1. Reddy C S, "Basic Structural Analysis", Tata McGraw-Hill Publishing Company Ltd.
2. Gupta S P, G S Pundit and R Gupta, "Theory of Structures", Vol II, Tata McGraw Hill Publications company Ltd.
3. V N Vazirani and M M Ratwani, "Analysis Of Structures", Vol. 2, Khanna Publishers
4. Wang C K, "Intermediate Structural Analysis", McGraw Hill, International Students Edition.
5. S.Rajasekaran and G. Sankarasubramanian, "Computational Structural Mechanics", PHI Learning Pvt. Ltd.,

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||Jai Sri Gurudev||

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Department of Civil Engineering

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Subject Name- Analysis of Indeterminate Structures

Semester: V

Subject Code- 15CV52

Academic Year: 2018-19

Faculty Name- Gagan Krishna R R

Course Outcomes:

1. Able to use slope deflection method for various civil engineering elements
2. Ability to calculating bending moment and shear force using moment distribution method and Kani's method.
3. Analyse the structural elements by using system stiffness method and system flexibility method.

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|| Jai Sri Gurudev ||
Adichunchanagiri Shikshana Trust (R)
BGS INSTITUTE OF TECHNOLOGY
Department of Civil Engineering

CO-PO & CO-PSO Mapping (15 Scheme)

Programme	Course Code	Course Name	Credits	L-T-P	Assessment		Exam Duration
					SEE	CIE	
B.E	16C302	Analysis of Indeterminate Structures	4	4-0-0	80	20	3Hrs

Course Outcomes (COs)

16C302.1	Able to use slope deflection method for various civil engineering elements.
16C302.2	Ability to calculating bending moment and shear force using moment distribution method and Kani's method.
16C302.3	Analyse the structural elements by using system stiffness method and system flexibility method.

Program Specific Outcomes (PSOs)


PSO 1: Graduates will be able to analyze, design and execute the civil engineering structures effectively for the sustainable development.

PSO 2: Graduates will acquire critical thinking abilities and technical skills for the usage of modern tools in development of civil engineering structures.

PSO 3: Graduates will be able to get opportunities for their professional growth, demonstrate communication and aptitude skills to face the challenges and needs of our society.

CO/PO/ PSO	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12	PSO 1	PSO 2	PSO 3
16C302.1	3	3	-	-	-	2	-	-	2	-	2	1	3	-	2
16C302.2	3	3	-	-	-	2	-	-	2	-	2	1	3	-	2
16C302.3	3	3	-	-	-	2	-	-	2	-	2	1	3	-	2
16C302	3	3	-	-	-	2	-	-	2	-	2	1	3	-	2


Course Owner


HOD
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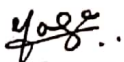
Faculty Name- Gagan Krishna R R


Course Outcomes:

1. Able to use slope deflection method for various civil engineering elements
2. Ability to calculating bending moment and shear force using moment distribution method and Kani's method.
3. Analyse the structural elements by using system stiffness method and system flexibility method.

CO Attainment

CO	CO 1	CO 2	CO 3
Attainment	1.19	1.34	1.95
Percentage (%)	39.66	44.66	65.00
Target (%)	60		
Faculty Comments	1. It is observed that CO 1 and CO 2 attainment values are less than target value. 2. Since the analysis procedure of MDM and SDM is difficult, CO 1 and CO 2 attainment is less		
Remarks by HOD	CO1 and CO2 attainment is low. To be improved.		


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Period : From 01-08-2018 To 30-11-2018

Semester : Odd / Even

Name of the Teacher : Lagan Krishna. R. R.
 Designation : Asst. Professor
 Department : Civil

Sl. No.	Sem. / Sec. / Branch	Subject	Code
1	<u>V Semester. B. E</u>	<u>Analysis of Indeterminate structures</u>	<u>15CV52</u>
2			
3			
4			

	REVIEWS at the End of the				End of Semester
	1st Month	2nd Month	3rd Month	4th Month	
Staff	<u>[Signature]</u>	<u>[Signature]</u>	<u>[Signature]</u>	<u>[Signature]</u>	<u>[Signature]</u>
HOD Reviewer	<u>[Signature]</u>	<u>[Signature]</u>	<u>[Signature]</u>	<u>[Signature]</u>	<u>[Signature]</u>

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LESSON PLANNING

Class : V Sem B. E

Subject with Code : Analysis of Indeterminate Structures [15CV52]

Period : From 01-08-2018 To 30-11-2018

1st Month : From 01-08-2018 To 31-08-2018	No. of Hours Planned :	24	Actual No. of Hours Taken :	21
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Chapter / Experiment / Module No. :

Module-1 : Slope deflection method.
 Analysis of Continuous beams
 Analysis of orthogonal rigid plane frames - 10 + 6 hours

Module-2 : Moment distribution method.
 Analysis of Continuous beams - 8 hours.

HOD's Review and Sign. :

2nd Month : From 01-09-2018 To 30-09-2018	No. of Hours Planned :	22	Actual No. of Hours Taken :	19
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Chapter / Experiment / Module No. :

Module-2 : Moment distribution method.
 Analysis of orthogonal rigid plane frames - 2 + 6 hours

Module-3 : Kani's method.
 Analysis of Continuous beams
 Analysis of orthogonal rigid plane frames - 10 + 4 hours

HOD's Review and Sign. :

3rd Month : From 01-10-2018 To 31-10-2018	No. of Hours Planned :	20	Actual No. of Hours Taken :	13
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Chapter / Experiment / Module No. :

Module-3 : Kani's method.
 Analysis of orthogonal rigid plane frames - 3 hours.

Module-4 : Matrix method of Analysis [Flexibility method]
 Analysis of Continuous beams
 Analysis of plane trusses
 Analysis of simple orthogonal rigid frames - 10 + 7 hours

HOD's Review and Sign. :

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4th Month : From 01-11-2018 To 30-11-2018	No. of Hours Planned : 17	Actual No. of Hours Taken : 07
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Chapter / Experiment / Module No. :

Module-5 : Matrix method of Analysis [Stiffness method]

Analysis of continuous beams

Analysis of plane trusses

Analysis of simple orthogonal rigid frames.

- 17 hours.

HOD's Review and Sign. :



5th Month : From To	No. of Hours Planned :	Actual No. of Hours Taken :
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Chapter / Experiment / Module No. :

HOD's Review and Sign. :

Tutorials & Tests Conducted on

Tutorial - 1 :

d	d	m	m	y	y	y	y
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Test - 1 :

0	8	0	9	2	0	1	8
---	---	---	---	---	---	---	---

Tutorial - 2 :

d	d	m	m	y	y	y	y
---	---	---	---	---	---	---	---

Test - 2 :

1	5	1	0	2	0	1	8
---	---	---	---	---	---	---	---

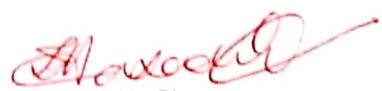
Tutorial - 3 :

d	d	m	m	y	y	y	y
---	---	---	---	---	---	---	---

Test - 3 :

2	2	1	1	2	0	1	8
---	---	---	---	---	---	---	---


Staff Sign


HOD's Sign
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B G S Institute of Technology,
B G Nagara - 571448**

MODULE 1 :- Slope Deflection Method.

Sign Conventions :-

1. Reactions.

$$\Sigma V = 0$$

↑ +ve

↓ -ve.

$$\Sigma H = 0$$

→ +ve

← -ve.

$$\Sigma M = 0$$

↻ -ve

↺ +ve.

2. Shear force.

From left to right

↑ +ve

↓ -ve

From right to left

↑ +ve

↓ +ve.

3. Bending Moment.

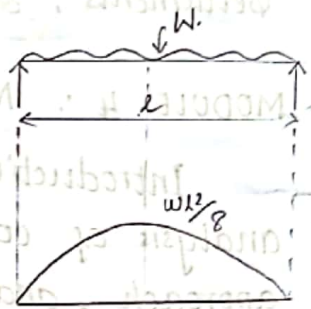
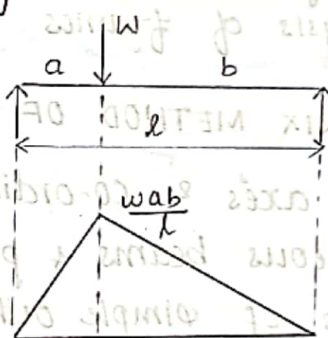
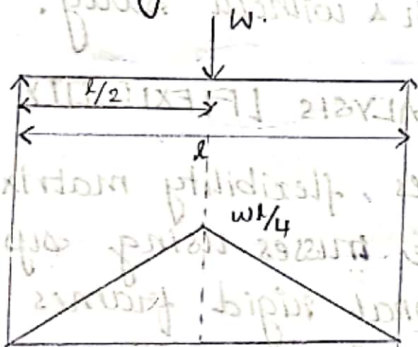
↻ +ve

↺ -ve.

Clockwise

Anti-clockwise.

4. Bending Moment diagrams.



Slope- Deflection Method.

This method is ideally suitable for the analysis of continuous beams & rigid jointed frames using this method basic unknowns like slopes & deflections of joints can be calculated.

Assumptions :-

- ⇒ All joints are rigid i.e., the angle b/w any 2 members in a joint does not change even after deformation due to loading.
- ⇒ The joints are assumed to rotate as a whole.

⇒ Distortions due to axial deformations are neglected because they are negligible or small. (3)

⇒ APPLICATIONS.

⇒ The method is illustrated by applying it to the following type of structures.

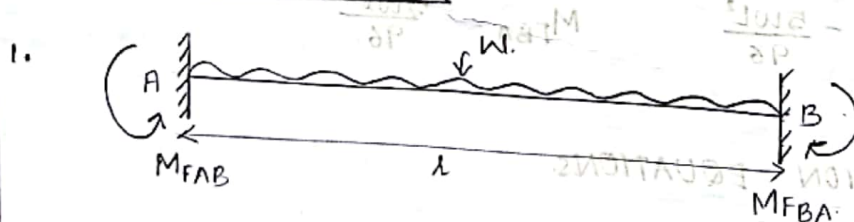
- * Continuous Beams.
- * Frames without side sway.
- * Frames with side sway.

STEPS TO BE FOLLOWED IN THE ANALYSIS OF CONTINUOUS BEAM BY SLOPE - DEFLECTION METHOD.

1. Each span of the continuous beam is taken as fixed beam & fixed end moments are noted down.
2. Using slope - deflection equations write all the end moments, in these some of the rotations & deflections will be unknown.
3. Write the joint - equilibrium equation.
4. Solve the joint - equilibrium eqⁿ to get the unknown rotations & deflections.
5. Substituting the values of unknowns in slope - deflection eqⁿ to determine the end moments.
6. Treating each member of the continuous beam as a simply supported beam subjected to a given loading & end moment determine the end moment & draw S.F.D & B.M.D.

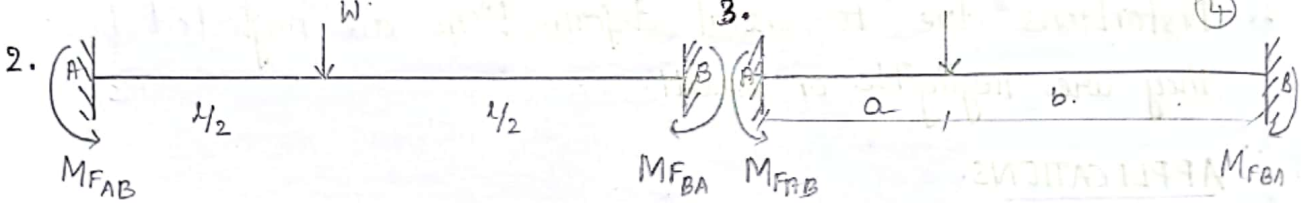
FORMULAE :-

I. FIXED END MOMENTS.



$$M_{FAB} = -\frac{wL^2}{12}$$

$$M_{FBA} = +\frac{wL^2}{12}$$

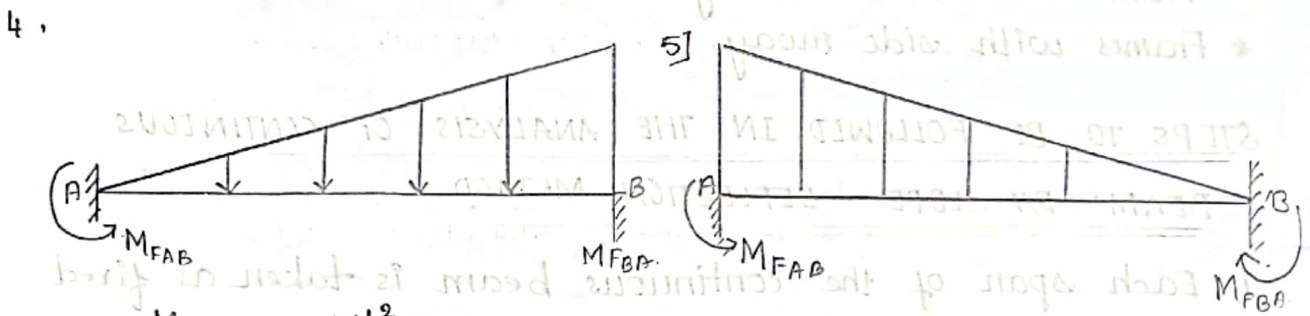


$$M_{FAB} = -\frac{wl}{8}$$

$$M_{FBA} = +\frac{wl}{8}$$

$$M_{FAB} = -\frac{wab^2}{l^2}$$

$$M_{FBA} = \frac{wa^2b}{l^2}$$

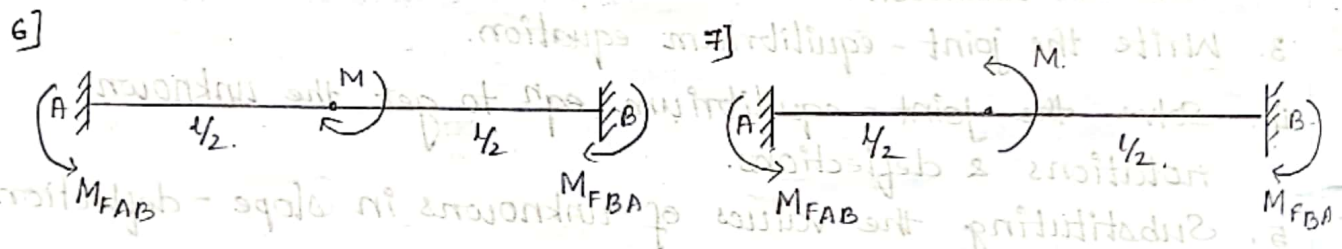


$$M_{FAB} = -\frac{wl^2}{30}$$

$$M_{FBA} = \frac{wl^2}{20}$$

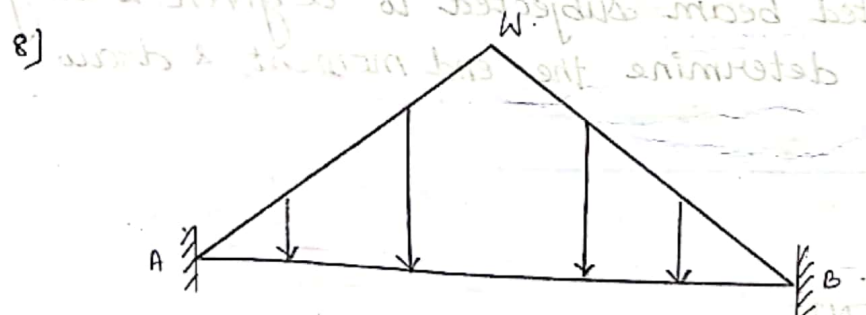
$$M_{FAB} = -\frac{wl^2}{20}$$

$$M_{FBA} = \frac{wl^2}{30}$$



$$M_{FAB} = M_{FBA} = \frac{M}{4}$$

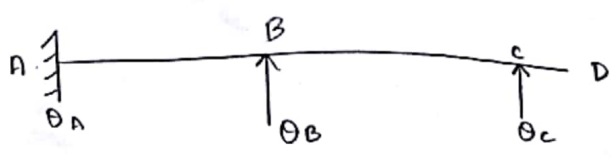
$$M_{FAB} = M_{FBA} = -\frac{M}{4}$$



$$M_{FAB} = -\frac{5wL^2}{96}$$

$$M_{FBA} = -\frac{5wL^2}{96}$$

II. SLOPE - DEFLECTION EQUATIONS.



$$M_{AB} = M_{FAB} + \frac{2EI}{L} \left[2\theta_A + \theta_B - \frac{3\delta}{L} \right]$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L} \left[2\theta_B + \theta_A - \frac{3\delta}{L} \right]$$

where θ = slope or rotation.

δ = Sinking or settlement.

EI = Flexural rigidity.

For the given beam, one side of the beam is fixed i.e., at A. so the value of θ becomes zero, i.e., $\theta_A = 0$.

III. EQUILIBRIUM CONDITIONS:

At intermediate supports sum of moments is equal to zero.

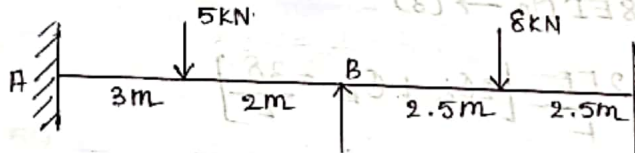
At 'B', $M_{BA} + M_{BC} = 0$.

At 'C', $M_{CB} + M_{CD} = 0$.

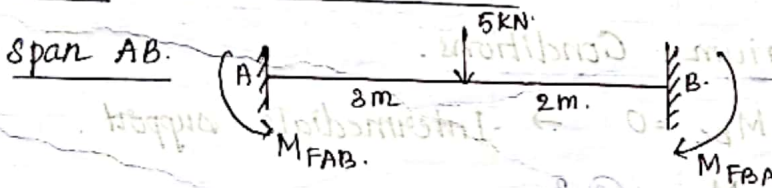
At the last support, moment = Zero
If it is simple, hinged or roller

PROBLEMS:-

1] Analyze the given continuous beam by using slope deflection method assume $EI = \text{constant}$ & draw SFD & BMD.



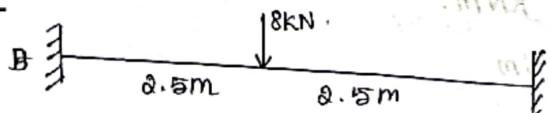
Soln: Step 1: Fixed end moments



$$M_{FAB} = -\frac{wab^2}{L^2} = -\frac{5 \times 3 \times (2)^2}{5^2} \Rightarrow M_{FAB} = -2.4 \text{ kNm}$$

$$M_{FBA} = \frac{wa^2b}{L^2} = \frac{5 \times (3)^2 \times 2}{5^2} \Rightarrow M_{FBA} = 3.6 \text{ kNm}$$

Span BC.



$$M_{FBC} = -\frac{wl}{8} = -\frac{8 \times 5}{8} \Rightarrow M_{FBC} = -5 \text{ kNm}$$

$$M_{FCB} = \frac{wl}{8} = \frac{8 \times 5}{8} \Rightarrow M_{FCB} = 5 \text{ kNm}$$

STEP 2: Slope - deflection Equation

$$\theta_A = \theta_C = 0 \rightarrow \text{Fixed supports}$$

$$\delta = 0 \rightarrow \text{NO sinking.}$$

For span AB.

$$M_{AB} = M_{FAB} + \frac{2EI}{L} \left[2\theta_A + \theta_B - \frac{3\delta}{L} \right]$$

$$M_{AB} = -2.4 + \frac{2EI}{L} [0 + \theta_B - 0]$$

$$M_{AB} = -2.4 + \frac{2EI}{L} (\theta_B)$$

$$M_{AB} = -2.4 + 0.4 EI \theta_B \rightarrow (1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L} \left[2\theta_B + \theta_A - \frac{3\delta}{L} \right]$$

$$= 3.6 + \frac{2EI}{L} (2\theta_B)$$

$$M_{BA} = 3.6 + 0.8 EI \theta_B \rightarrow (2)$$

For span BC

$$M_{BC} = M_{FBC} + \frac{2EI}{L} \left[2\theta_B + \theta_C - \frac{3\delta}{L} \right]$$

$$= -5 + \frac{2EI}{L} [2\theta_B + 0 - 0]$$

$$M_{BC} = -5 + 0.8 EI \theta_B \rightarrow (3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L} \left[2\theta_C + \theta_B - \frac{3\delta}{L} \right]$$

$$M_{CB} = 5 + 0.4 EI \theta_B \rightarrow (4)$$

STEP 3: Equilibrium Conditions.

① + B $M_{BA} + M_{BC} = 0 \rightarrow$ Intermediate support.

Substitute $M_{BA} + M_{BC}$ @ B.

$$3.6 + 0.8 EI \theta_B + [-5 + 0.8 EI \theta_B] = 0$$

$$\theta_B = 0.875 EI$$

STEP 4: Substitute eqn (5) in (1)

$$M_{AB} = -2.05 \text{ KNm.}$$

$$M_{BA} = 4.3 \text{ KNm}$$

$$M_{BC} = -4.3 \text{ KNm}$$

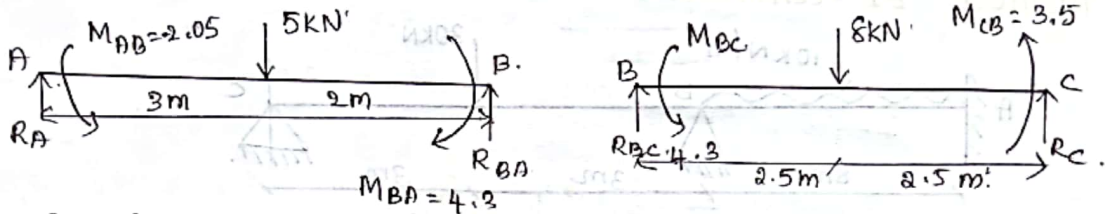
$$M_{CB} = 5.35 \text{ KNm}$$

$$M_{AB} = -2.4 + 0.4 EI \times 0.875 \times EI$$

STEP 5: SFD.

(7)

Span AB.



$\Sigma V = 0 \quad \uparrow +ve.$

$\Sigma H = 0 \quad \rightarrow +ve$

$\Sigma M = 0 \quad \curvearrowright +ve$

$\Sigma M_A = 0;$

$-2.05 + 4.3 - R_{BA} \cdot 5 + 5 \times 3 = 0$

$R_{BA} = 3.45$

$\Sigma V = 0;$

$R_A + R_B - 5 = 0$

$R_A = 1.55 \text{ kN.}$

$\Sigma M_B = 0;$

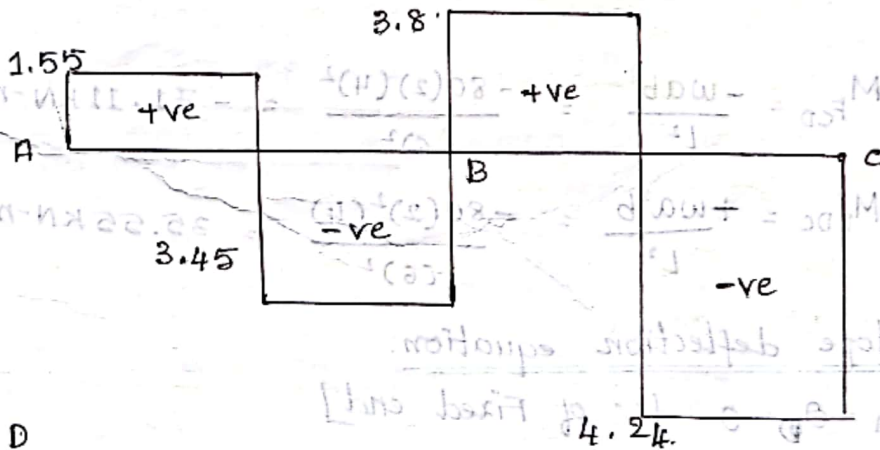
$-4.35 - R_C \times 4.8 \times 0.5 - 5.35 = 0.$

$R_C = 4.2 \text{ kN.}$

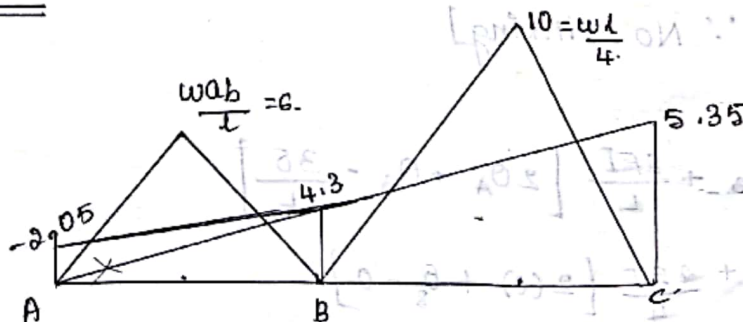
$\Sigma V = 0; \quad R_{BC} + R_C - 8 = 0.$

$R_{BC} = 3.8 \text{ kN.}$

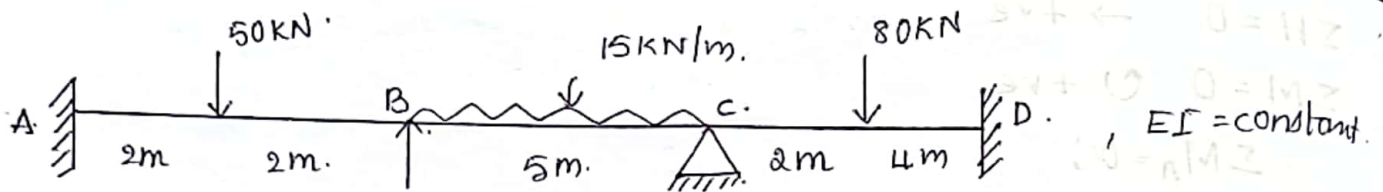
SFD.



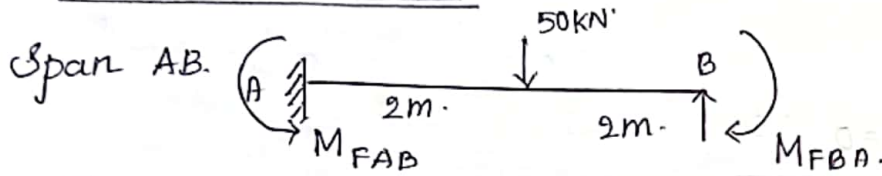
B.M.D



3]



STEP 1: Fixed end moments



$$M_{FAB} = \frac{-wL}{8} = \frac{-50(4)}{8} = -25 \text{ KN-m}$$

$$M_{FBA} = \frac{wL}{8} = \frac{50(4)}{8} = 25 \text{ KN-m}$$

Span BC,

$$M_{FBC} = \frac{-wL^2}{12} = \frac{-15(5)^2}{12} = \boxed{M_{FBC} = -31.25 \text{ KN-m}}$$

$$M_{FCB} = \frac{wL^2}{12} = \frac{15(5)^2}{12} = \boxed{M_{FCB} = 31.25 \text{ KN-m}}$$

Span CD,

$$M_{FCD} = \frac{-wab^2}{L^2} = \frac{-80(2)(4)^2}{(6)^2} = -71.11 \text{ KN-m}$$

$$M_{FDC} = \frac{+wa^2b}{L^2} = \frac{-80(2)^2(4)}{(6)^2} = 35.55 \text{ KN-m}$$

STEP 2: Slope deflection equation.

$$\theta_A = \theta_D = 0 \quad [\because \text{of Fixed end}]$$

$$\delta = 0 \quad [\because \text{No sinking}]$$

For span AB:

$$M_{AB} = M_{FAB} + \frac{2EI}{L} \left[2\theta_A + \theta_B - \frac{3\delta}{L} \right]$$

$$= -25 + \frac{2EI}{4} \left[2(0) + \theta_B - 0 \right]$$

$$M_{AB} = -25 + 0.5EI\theta_B \rightarrow (1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L} \left[2\theta_B + \theta_A - \frac{3\delta}{L} \right] \quad (2)$$

$$= 25 + 0.5EI [2\theta_B]$$

$$= 25 + EI \cdot \theta_B \rightarrow (2)$$

For span BC

$$M_{BC} = M_{FBC} + \frac{2EI}{L} \left[2\theta_B + \theta_C - \frac{3\delta}{L} \right]$$

$$= -31.25 + 0.8EI\theta_B + 0.4\theta_C EI \rightarrow (3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L} \left[2\theta_C + \theta_B - \frac{3\delta}{L} \right]$$

$$= 31.25 + 0.8EI\theta_C + 0.4EI\theta_B \rightarrow (4)$$

For Span CD,

$$M_{CD} = M_{FCD} + \frac{2EI}{L} \left[2\theta_C + \theta_D - \frac{3\delta}{L} \right]$$

$$= -71.11 + 0.66EI\theta_C \rightarrow (5)$$

$$M_{DC} = M_{FDC} + \frac{2EI}{L} \left[2\theta_D + \theta_C - \frac{3\delta}{L} \right]$$

$$= 35.55 + 0.33EI\theta_C \rightarrow (6)$$

STEP 3:- Equilibrium Conditions.

B & C are the intermediate supports \therefore

Σ Moments @ the joints = 0

@ B $\therefore M_{BA} + M_{BC} = 0$

$$25 + EI \cdot \theta_B + (-31.25 + 0.8EI\theta_B + 0.4\theta_C \cdot EI) = 0$$

$$25 + EI \cdot \theta_B - 31.25 + 0.8EI\theta_B + 0.4EI\theta_C = 0$$

$$-6.25 + 1.8EI\theta_B + 0.4EI\theta_C = 0 \rightarrow (7)$$

@ C: $M_{CB} + M_{CD} = 0$

$$31.25 + 0.8EI\theta_C + 0.4EI\theta_B - 71.11 + 0.66EI\theta_C = 0$$

$$-39.86 + 0.4EI\theta_B + 1.46EI\theta_C = 0 \rightarrow (8)$$

By solving (7) & (8)

$$\theta_B = \frac{-2.76}{EI}, \quad \theta_C = \frac{28.05}{EI}$$

STEP 4: Final Bending Moments - Sub the values of θ_B & θ_C

in eqns 1-6

$$M_{AB} = -25 + 0.5EI\theta_B = -26.38$$

$$M_{BA} = 25 + EI \cdot \left[\frac{-2.76}{EI} \right] = 22.24$$

$$M_{BC} = -31.25 + 0.8EI\theta_c + 0.4EI\theta_B$$

$$= -31.25 + 0.8EI\left(\frac{28.05}{EI}\right) + 0.4EI\left(\frac{-2.76}{EI}\right)$$

$$= -22.238 \text{ KN-m}$$

(10)

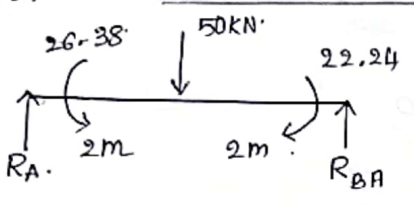
$$M_{CB} = 31.25 + 0.8EI\theta_c + 0.4EI\theta_B$$

$$= 52.58 \text{ KN-m}$$

$$M_{CD} = -71.11 + 0.66EI\theta_c = -52.59 \text{ KN-m}$$

$$M_{DC} = 35.55 + 0.33EI\theta_c = 44.80 \text{ KN-m}$$

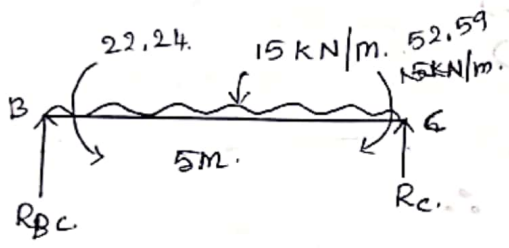
STEP 5: Draw SFD & BMD.



$$\sum M_A = 0;$$

$$4(-R_{BA}) + 22.24 + 50(2) - 26.38 = 0$$

$$R_{BA} = 23.965 \text{ KN}$$



$$\sum V = 0;$$

$$R_A + 23.965 - 50 = 0$$

$$R_A = 26.035 \text{ KN}$$

$$\sum M_B = 0;$$

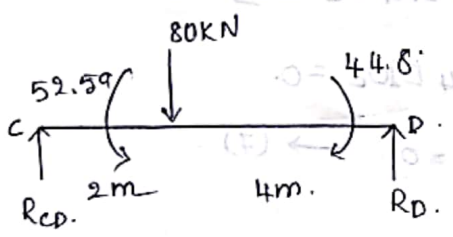
$$(-R_C)5 + 15(5)(2.5) - 22.24 + (15 \times 5)(2.5) = 0$$

$$R_C = 43.07 \text{ KN}$$

$$\sum V = 0;$$

$$R_{BC} + 43.07 - 15(5) = 0$$

$$R_{BC} = 31.93 \text{ KN}$$



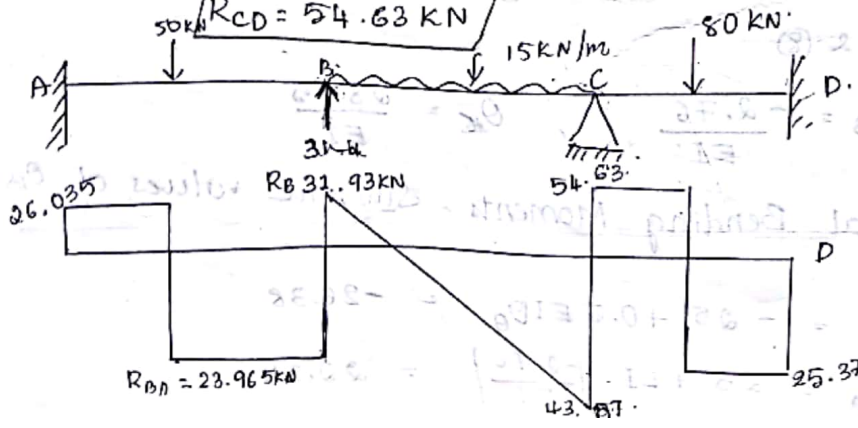
$$\sum M_C = 0;$$

$$-R_D(6) + 44.8 + 80(2) - 52.59 = 0$$

$$R_D = 25.36 \text{ KN}$$

$$\sum V = 0; 25.36 - 80 + R_{CD} = 0$$

$$R_{CD} = 54.63 \text{ KN}$$



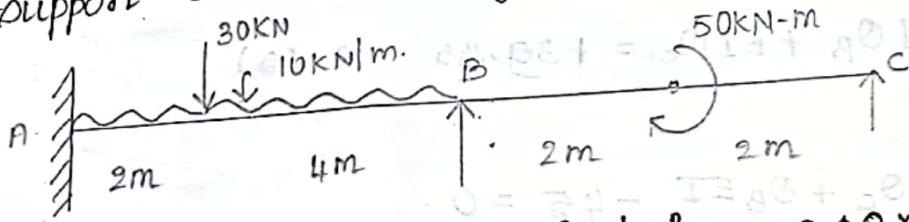
$$\frac{wL}{4} = \frac{50 \times 4}{4} = 50$$

$$\frac{wL^2}{8} = \frac{15 \times 5^2}{8} = 46.875$$

$$\frac{wab}{l} = \frac{80 \times 2 \times 4}{6} = 106.66$$

Sinking of Supports :-

1) Analyse the beam shown by slope-deflection method & draw S.F.D, B.M.D & elastic curve. The beam sinks by 5mm. The support 'B' sinks by 5mm. Take $E = 210 \text{ GPa}$, $I = 0.1 \times 10^9 \text{ mm}^4$.



Soln :- $E = 210 \text{ GPa} = 210 \times 10^9 \text{ N/m}^2 = 210 \times 10^3 \text{ N/mm}^2$
 $EI = 210 \times 10^3 \times 0.1 \times 10^9 = 2.1 \times 10^{13} \text{ N-mm}^2$
 $EI = \frac{2.1 \times 10^{13}}{(1000)(1000)^2} = 2.1 \times 10^4 \text{ KN-m}^2$

STEP 1: Fixed end moments.

Span AB :- $M_{FAB} = \frac{-wab^2}{L^2} - \frac{wL^2}{12} = \frac{-30 \times 2 \times (4)^2}{(6)^2} - \frac{10 \times (6)^2}{12}$

$$M_{FAB} = -56.66 \text{ KN-m.}$$

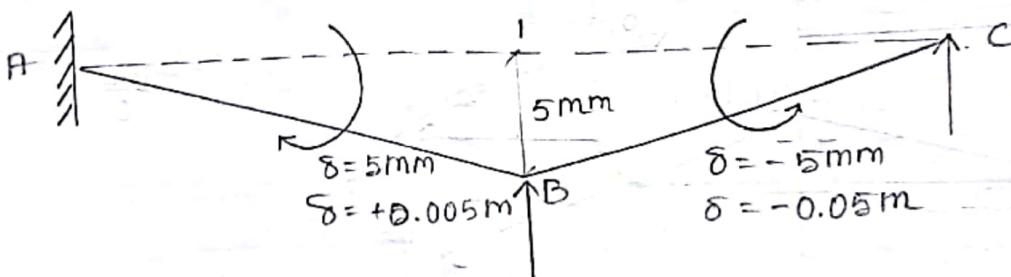
$$M_{FBA} = + \frac{wa^2b}{L^2} + \frac{wL^2}{12}$$

$$M_{FBA} = 43.33 \text{ KN-m}$$

Span BC : $M_{FBC} = M_{FCB} = \frac{M}{4} = \frac{50}{4}$ [since clockwise +ve]

$$M_{FBC} = M_{FCB} = 12.5$$

STEP 2: slope-deflection method. :- $\theta_A = 0$, $\delta = 0.005 \text{ m}$



$$M_{AB} = M_{FAB} + \frac{2EI}{L} \left[2\theta_A + \theta_B - \frac{3\delta}{L} \right]$$

$$= -56.66 + \frac{2(2.1 \times 10^4)}{6} \left[\theta_B - \frac{3(0.005)}{6} \right]$$

$$\begin{aligned} \textcircled{17} &= -56.66 + 7000 \theta_B - 17.5 \\ &= -74.16 + 7000 \theta_B \rightarrow (1) \end{aligned}$$

$$\begin{aligned} M_{BA} &= M_{FBA} + \frac{2EI}{L} \left[2\theta_B + \theta_A - \frac{3\delta}{L} \right] \\ &= 43.33 + 14000 \theta_B - 17.5 \\ &= 25.83 + 14000 \theta_B \rightarrow (2) \end{aligned}$$

$$\begin{aligned} M_{BC} &= M_{FBC} + \frac{2EI}{L} \left[2\theta_B + \theta_C - \frac{3\delta}{L} \right] \\ &= 12.5 + \frac{2(2.1 \times 10^4)}{4} \left[2\theta_B + \theta_C + 3.75 \times 10^{-3} \right] \\ &= 12.5 + 10500 \theta_C + 21000 + 39.375 \\ &= 51.875 + 21000 \theta_B + 10500 \theta_C \rightarrow (3) \end{aligned}$$

$$\begin{aligned} M_{CB} &= M_{FCB} + \frac{2EI}{L} \left[2\theta_C + \theta_B - \frac{3\delta}{L} \right] \\ &= 12.5 + 21000 \theta_C + 10500 \theta_B + 39.375 \\ &= 51.875 + 21000 \theta_C + 10500 \theta_B \rightarrow (4) \end{aligned}$$

STEP 4:- Equilibrium conditions

$$M_{BA} + M_{BC} = 0;$$

$$25.83 + 14000 \theta_B + 51.875 + 21000 \theta_B + 10500 \theta_C = 0$$

$$\cancel{25.83 + 51.875} + 77.705 + 35000 \theta_B + 10500 \theta_C = 0 \rightarrow (5)$$

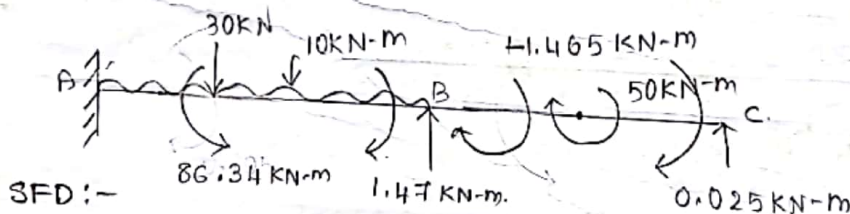
$$M_{CB} = 0; \quad 51.875 + 21000 \theta_C + 10500 \theta_B = 0 \rightarrow (6)$$

$$\theta_B = -1.74 \times 10^{-3}; \quad \theta_C = -1.600 \times 10^{-3}$$

$$M_{AB} = -86.34 \text{ KN-m} \quad M_{BA} = 1.47 \text{ KN-m}$$

$$M_{BC} = -1.465 \text{ KN-m} \quad M_{CB} = -0.025 \text{ KN-m} \approx 0$$

STEP 5: SFD & BMD



$$\Sigma M_A = 0; \quad -R_B \times 6 + 1.47 - 86.34 + 10 \times 6 \times \frac{6}{2} + 30 \times 2 = 0$$

$$\Sigma V = 0; \quad R_B = 25.855 \text{ KN}$$

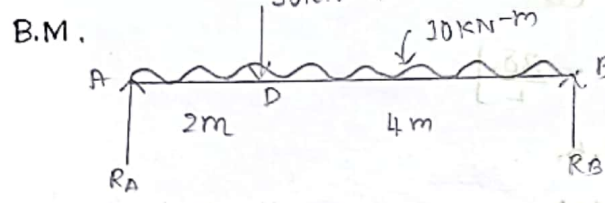
$$R_A = 64.15 \text{ KN}$$

$$\Sigma M_B = 0; \quad 4(-R_C) + 0.025 + 50 + 1.47 = 0$$

$$R_C = 12.87 \text{ KN}$$

$\Sigma V=0; R_B = -12.13 \text{ KN}$

$\Sigma M_A=0;$



$-R_B \times 6 + 10 \times 6 \times \frac{6}{2} + 30 \times 2 = 0$

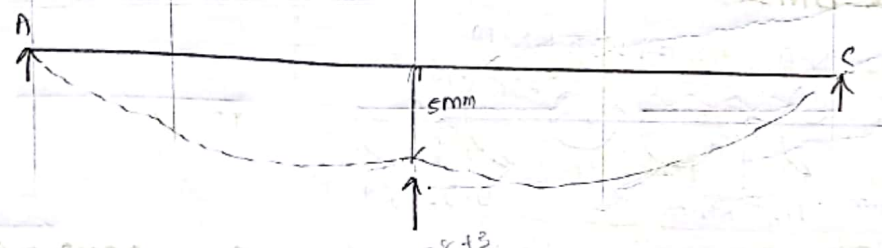
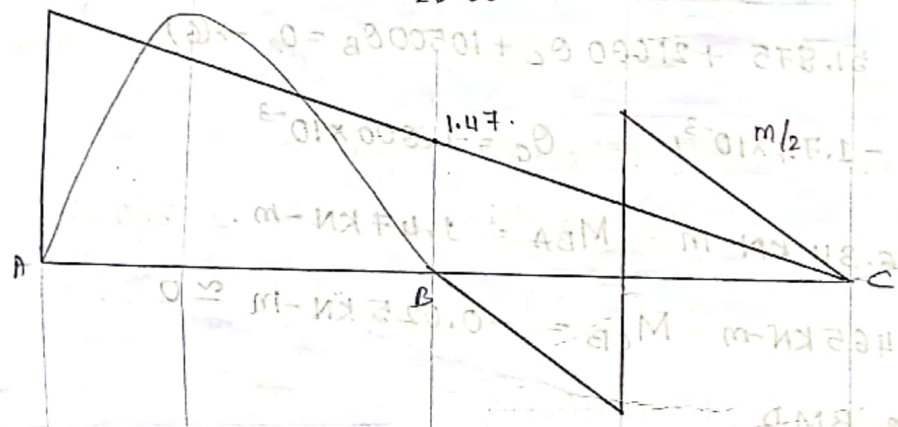
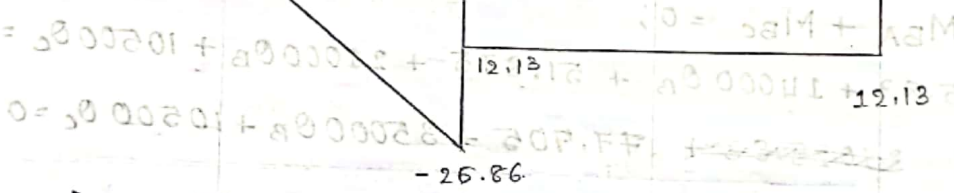
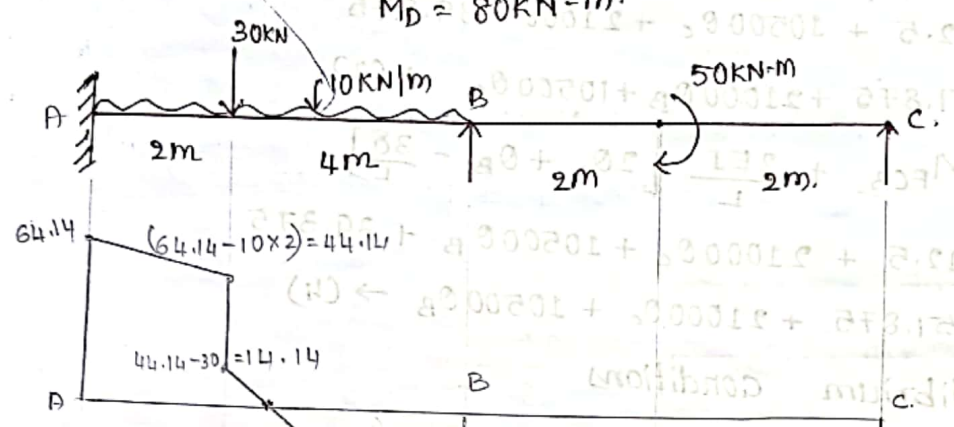
$R_B = 40 \text{ KN}$

$\Sigma V=0; R_A + 40 - 10 \times 6 - 30 = 0$

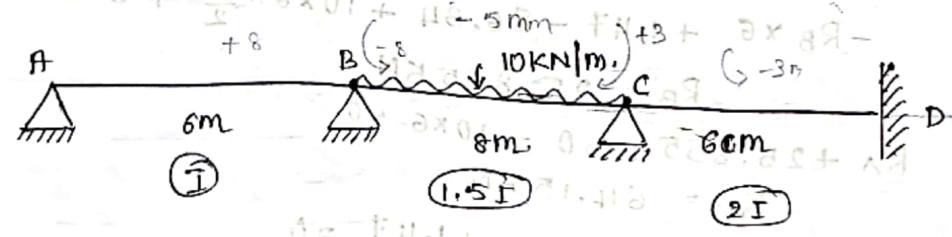
$R_A = 50 \text{ KN}$

$\Sigma M_D=0; 40 \times 4 - 10 \times 4 \times \frac{4}{2} = 0$

$M_D = 80 \text{ KN-m}$



2]



Supports B & C settles by 8mm & 3mm respectively. Take $EI = 2 \times 10^4 \text{ KN-m}^2$

SDP:- STEP 1: Fixed end Moments

$$M_{FAB} = 0$$

$$M_{FBA} = 0$$

Because of no loads.

(1)

$$M_{CD} = 0$$

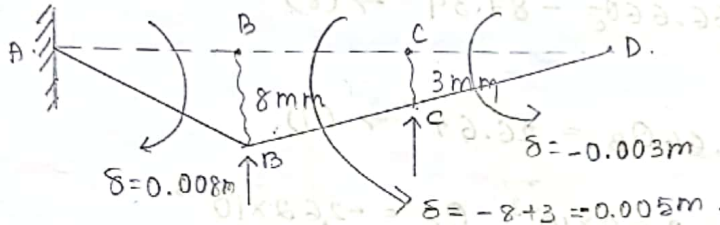
$$M_{DC} = 0$$

$$M_{FBC} = \frac{-wl^2}{12} = \frac{-10(8)^2}{12} \Rightarrow M_{FBC} = -53.33 \text{ KN-m}$$

$$M_{FCB} = 53.33 \text{ KN-m}$$

STEP 2: Slope - deflection eqn.

$$\theta_D = 0$$



$$\begin{aligned} M_{AB} &= M_{FAB} + \frac{2EI}{L} \left[2\theta_A + \theta_B - \frac{3\delta}{L} \right] \\ &= \frac{2(2 \times 10^4)}{6} \left[2\theta_A + \theta_B - \frac{3(0.008)}{6} \right] \\ &= 6666.66\theta_B + 13333.33\theta_A - 26.66 \rightarrow (1) \end{aligned}$$

$$6.666.66\theta_B + 13333.33\theta_A = 26.66 \rightarrow (1')$$

$$\begin{aligned} M_{BA} &= M_{FBA} + \frac{2EI}{L} \left[2\theta_B + \theta_A - \frac{3\delta}{L} \right] \\ &= 6666.66\theta_A + 13333.33\theta_B - 26.66 \rightarrow (2) \end{aligned}$$

$$\begin{aligned} M_{BC} &= M_{FBC} + \frac{2EI}{L} \left[2\theta_B + \theta_C - \frac{3\delta}{L} \right] \\ &= -53.33 + \frac{2(1.5)(2 \times 10^4)}{8} \left[2\theta_B + \theta_C + \frac{3(0.005)}{1.5} \right] \\ &= -53.33 + 7500\theta_C + 15000\theta_B + 14.0625 \\ &= -39.26 = 7553.32 + 15000\theta_B + 7500\theta_C \rightarrow (3) \end{aligned}$$

$$\begin{aligned} M_{CB} &= M_{FCB} + \frac{2EI}{L} \left[2\theta_C + \theta_B - \frac{3\delta}{L} \right] \\ &= 53.33 + 7500\theta_B + 15000\theta_C + 1.875 \times 10^{-3}(7500) \\ &= 15000\theta_C + 7500\theta_B + 67.39 \rightarrow (4) \end{aligned}$$

$$\begin{aligned} M_{CD} &= M_{FCD} + \frac{4EI}{L} \left[2\theta_C + \theta_D - \frac{3\delta}{L} \right] \\ &= 13333.33\theta_C - 26666.66\theta_C + 20 \rightarrow (5) \end{aligned}$$

$$M_{DC} = 13333.33\theta_C + 20 \rightarrow (6)$$

STEP 3: Equilibrium Conditions

$$M_{BA} + M_{BC} = 0$$

$$13333.33\theta_B + 6666.66\theta_A - 26.67 + 15000\theta_B + 7500\theta_C - 39.26 = 0$$

$$+ 65.93 = + 28333.33\theta_B + 6666.66\theta_A + 7500\theta_C = 0 \rightarrow (7)$$

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@ C $M_{CB} + M_{CD} = 0$

$$15000\theta_C + 7500\theta_B + 67.39 + 26666.66\theta_C + 20 = 0$$

$$7500\theta_B + 41666.66\theta_C - 87.39 \rightarrow (8)$$

$$M_{AB} = 0$$

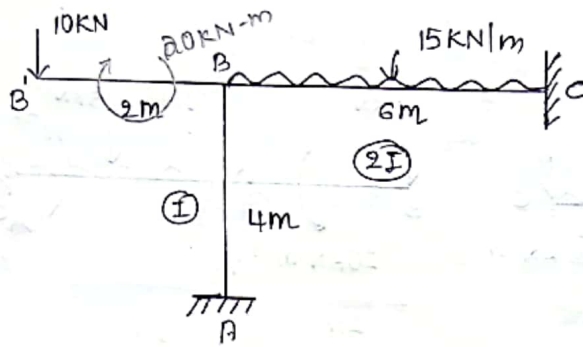
$$13333.33\theta_A + 6666.66\theta_B = 26.67 \rightarrow (9)$$

$$\theta_A = 5.57 \times 10^{-4} \quad \theta_B = 2.88 \times 10^{-3} - \theta_C = -2.62 \times 10^{-3}$$

$$M_{AB} = -0.043 \approx 0 \quad M_{BA} = 15.44 \text{ kN} \quad M_{BC} = +15.56$$

$$M_{CB} = 49.8 \quad M_{CD} = -49.8 \quad M_{DC} = -14.89$$

1) Analyse the frame as shown in fig by using slope-deflection method & draw SFD, B.MD & elastic curve. (21)



Force act in anticlockwise moment resist in clockwise hence +ve

Solⁿ: step 1: Fixed end moment.

$$M_{FAB} = 0 ; M_{FBA} = 0 ;$$

$$M_{FBC} = \frac{-WL^2}{12} = \frac{-15(6)^2}{12} \Rightarrow M_{FBC} = -45 \text{ kN-m}$$

$$M_{FCB} = 45 \text{ kN-m}$$

$$M_{BB'} = 10 \times 2 = +20 \text{ kN-m}$$

step 2: Slope-deflection method equation.

$$\theta_A = 0 ; \theta_C = 0 ; \delta = 0$$

There is no slope-deflection eqⁿ for overhanging BB'

$$M_{AB} = M_{FAB} + \frac{2EI}{L} \left[2\theta_A + \theta_B - \frac{3\delta}{L} \right]$$

$$= 0 + \frac{2EI}{4} \left[\theta_B \right]$$

$$= 0.5EI\theta_B \rightarrow (1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L} \left[2\theta_B + \theta_A - \frac{3\delta}{L} \right]$$

$$= EI\theta_B \rightarrow (2)$$

$$M_{BC} = M_{FBC} + \frac{4EI}{L} \left[2\theta_B + \theta_C - \frac{3\delta}{L} \right]$$

$$= -45 + \frac{4EI}{6} \left[2\theta_B \right]$$

$$= -45 + 1.333EI\theta_B \rightarrow (3)$$

$$M_{CB} = M_{FCB} + \frac{4EI}{L} \left[2\theta_C + \theta_B - \frac{3\delta}{L} \right]$$

$$= 45 + 0.666EI\theta_B \rightarrow (4)$$

STEP 3: Equilibrium conditions.

$$M_{BA} + M_{BC} + M_{BB'} = 0$$

$$EI\theta_B + (-45) + 1.333EI\theta_B + 20 = 0$$

$$\theta_B = \frac{10.73}{EI}$$

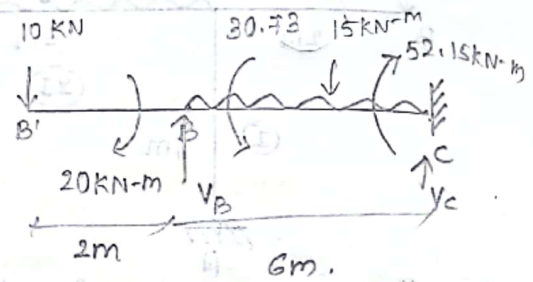
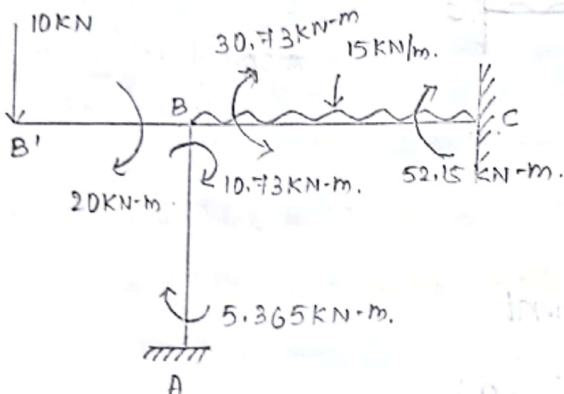
$M_{AB} = 5.365 \text{ KN-m}$

$M_{BA} = 10.73 \text{ KN-m}$

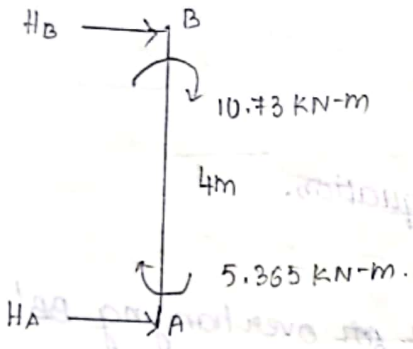
$M_{BC} = -30.7291 \text{ KN-m}$

$M_{CB} = 52.1 \text{ KN-m}$

$M_{BB'} = 20 \text{ KN-m}$



Horizontal Forces.



$\Sigma M_B = 0;$
 $-4H_A + 5.365 + 10.73 = 0$

$H_A = 4.023 \text{ KN}$

$\Sigma H = 0;$ $H_A + H_B = 0$

$H_B = -4.023 \text{ KN}$

$\Sigma M_C = 0;$
 $-10 \times 8 + 20 + 6V_B - 30.73 + 52.15 = 0$

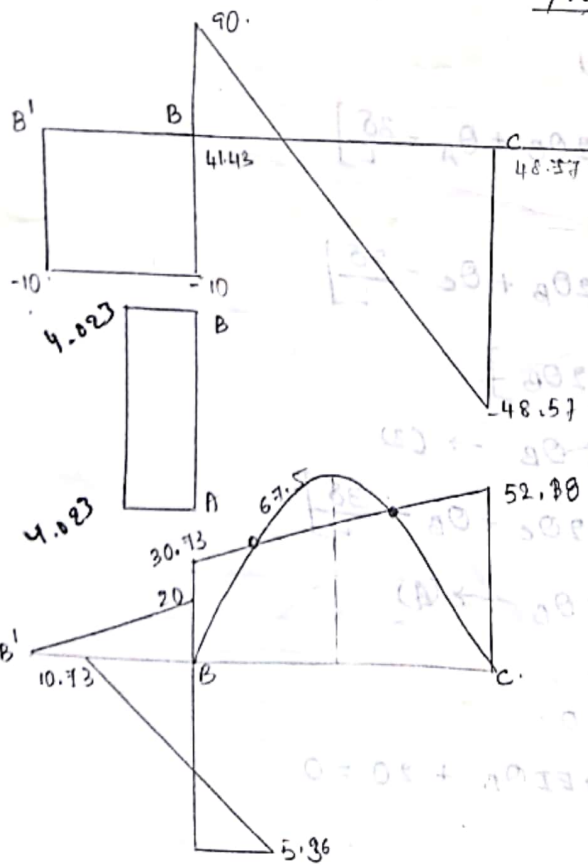
$15 \times 6 \times \frac{6}{2} = 0$

$V_B = 51.43 \text{ KN}$

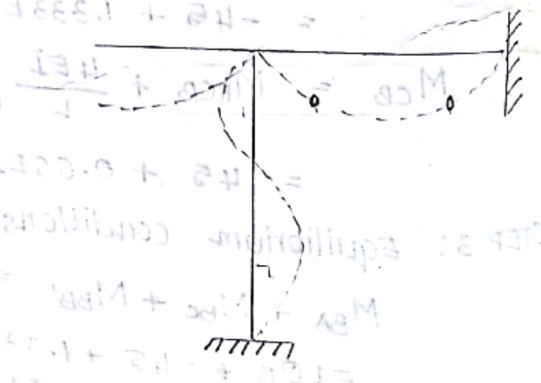
$\Sigma V = 0;$

$+10 + 51.43 - 15 \times 6 + V_C = 0$

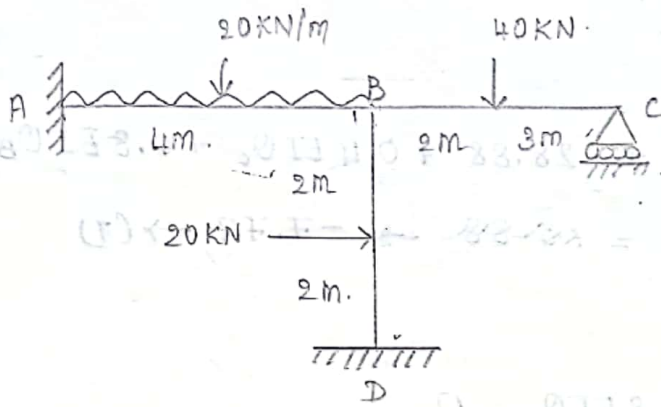
$V_C = 48.57 \text{ KN}$



-10
 $-10 + 51.43 = 41.43$
 $15 \times 6 = 90$
 $41.43 - 90 = -48.57$
 $-48.57 + 48.57 = 0$



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SOP: Step 1: Fixed end moment.

$$\text{Span AB, } M_{FAB} = \frac{-wL^2}{12} = \frac{-20(4)^2}{12} \Rightarrow M_{FAB} = -26.66 \text{ kN-m}$$

$$M_{FBA} = \frac{wL^2}{12} = \frac{20(4)^2}{12} \Rightarrow M_{FBA} = 26.66 \text{ kN-m}$$

$$\text{Span BC, } M_{FBC} = \frac{-wab^2}{L^2} = \frac{-40 \times 2 \times (3)^2}{(5)^2} \Rightarrow M_{FBC} = -28.8 \text{ kN-m}$$

$$M_{FCB} = \frac{+wa^2b}{L^2} = \frac{40 \times 4 \times 3}{(5)^2} \Rightarrow M_{FCB} = 19.2 \text{ kN-m}$$

$$\text{Span BD, } M_{FBD} = \frac{+WL}{8} = \frac{+20(4)}{8} \Rightarrow M_{FBD} = 10 \text{ kN-m}$$

$$M_{FDB} = \frac{-WL}{8} = \frac{-20(4)}{8} \Rightarrow M_{FDB} = -10 \text{ kN-m}$$

Step 2: Slope-deflection equations.

$$\theta_A = 0 ; \theta_D = 0 ; \delta = 0$$

$$M_{AB} = M_{FAB} + \frac{2EI}{L} \left[2\theta_A + \theta_B - \frac{3\delta}{L} \right]$$

$$= -26.66 + \frac{2EI}{4} [\theta_B]$$

$$= -26.66 + 0.5EI\theta_B \rightarrow (1)$$

$$M_{BA} = -M_{FBA} + \frac{2EI}{L} \left[2\theta_B + \theta_A - \frac{3\delta}{L} \right]$$

$$= 26.66 + EI\theta_B \rightarrow (2)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L} \left[2\theta_B + \theta_C - \frac{3\delta}{L} \right]$$

$$= -28.88 + 0.4EI\theta_C + 0.8EI\theta_B \rightarrow (3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L} \left[2\theta_C + \theta_B - \frac{3\delta}{L} \right]$$

$$= 19.2 + 0.4EI\theta_B + 0.8EI\theta_C \rightarrow (4)$$

$$M_{BBD} = M_{FBD} + \frac{2EI}{L} \left[2\theta_B + \theta_D - \frac{3\delta}{L} \right] \quad (5)$$

$$= 10 + 0.5EI\theta_D + EI\theta_B \rightarrow (5)$$

$$M_{DB} = M_{FDB} + \frac{2EI}{L} \left[2\theta_D + \theta_B - \frac{3\delta}{L} \right]$$

$$= -10 + 0.5EI\theta_B \rightarrow (6)$$

Step 3: Equilibrium Conditions.

$$M_{BD} + M_{BA} + M_{BC} = 0;$$

$$10 + EI\theta_B + 26.67 + EI\theta_B = -28.88 + 0.4EI\theta_C + 0.8EI\theta_B$$

$$2.8EI\theta_B + 0.4EI\theta_C = -10.88 \rightarrow -7.79 \rightarrow (7)$$

$$M_{CB} = 0;$$

$$19.2 + 0.4EI\theta_B + 0.8EI\theta_C = 0$$

$$0.4EI\theta_B + 0.8EI\theta_C = -19.2 \rightarrow (8)$$

$$\theta_B = \frac{0.69}{EI}$$

$$\theta_C = \frac{-24.34}{EI}$$

$$M_{AB} = -26.34 \text{ KN-m}$$

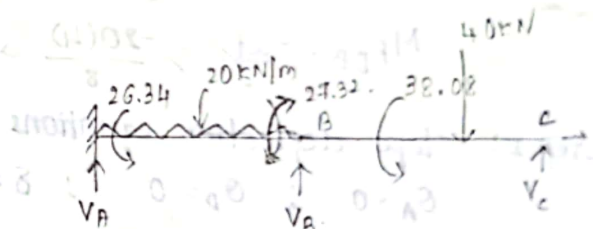
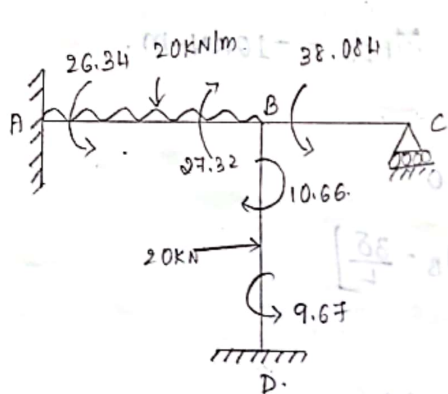
$$M_{BA} = 27.32 \text{ KN-m}$$

$$M_{BC} = -38.084 \text{ KN-m}$$

$$M_{CB} = 0$$

$$M_{BD} = 10.66 \text{ KN-m}$$

$$M_{DB} = -9.67 \text{ KN-m}$$



$$\sum M_A = 0; \text{ Span (AB)}$$

$$-4V_B - 26.34 + 20 \times 4 \times \frac{4}{2} + 27.32 = 0$$

$$V_B = 40.245 \text{ KN}$$

$$\sum V = 0;$$

$$-V_A + 40.245 - 20 \times 4 = 0$$

$$V_A = 39.755 \text{ KN}$$

Span BC;

$$\sum M_B = 0;$$

$$-40 + 5V_C - 38.08 = 0$$

$$V_C = 8.4$$

$$V_B = 31.6$$

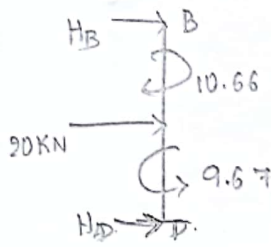
B.M

$$\frac{wL^2}{8} = 48$$

$$\frac{wL}{4} = 20$$

$$\frac{wL^2}{8} = 40$$

span BD



$$\sum M_B = 0;$$

$$-4H_D - 20 \times 2 - 9.67 + 10.66 = 0$$

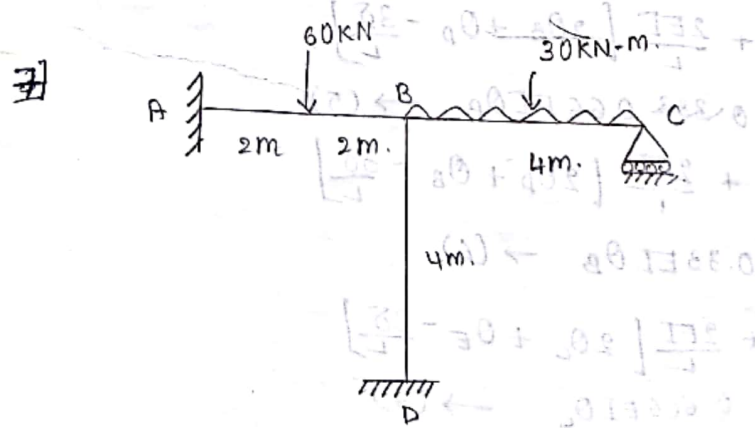
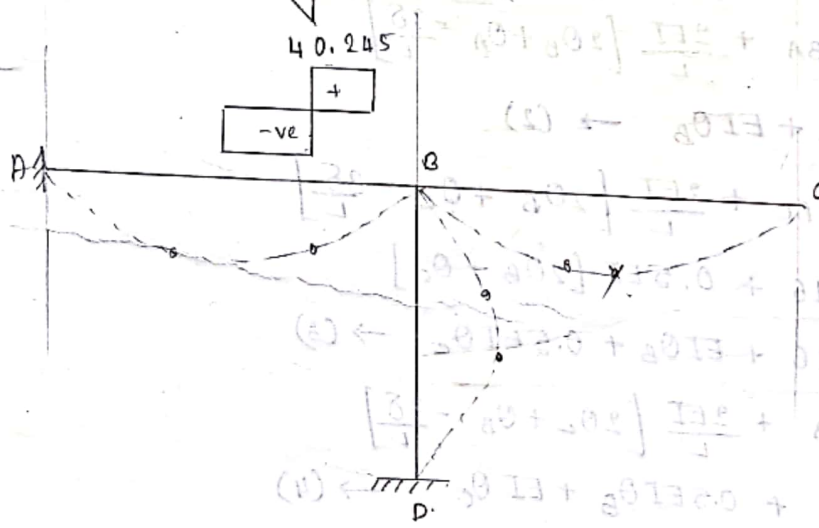
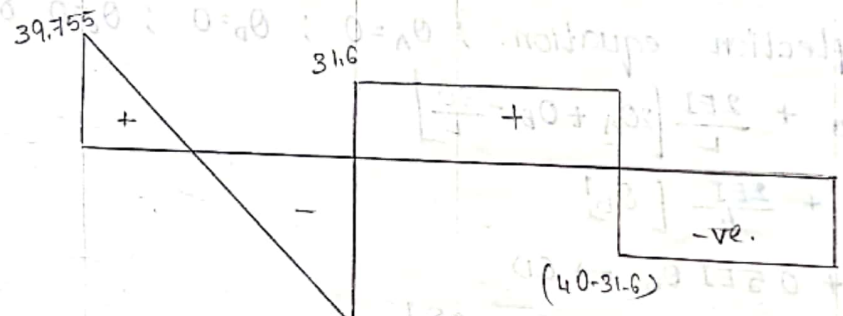
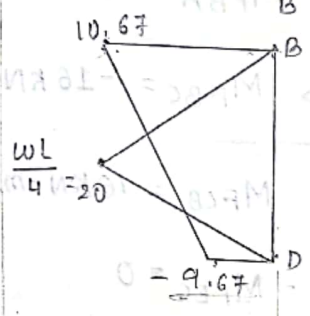
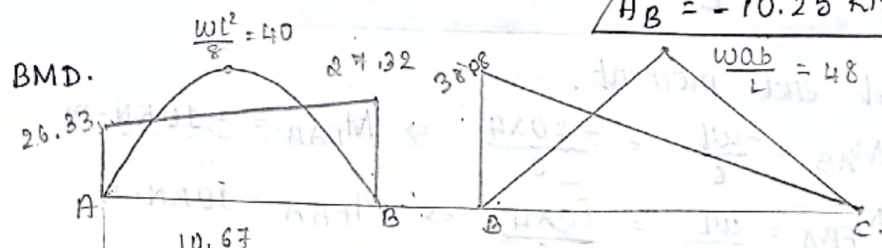
$$H_D = -9.752 \text{ kN}$$

$$\sum H = 0;$$

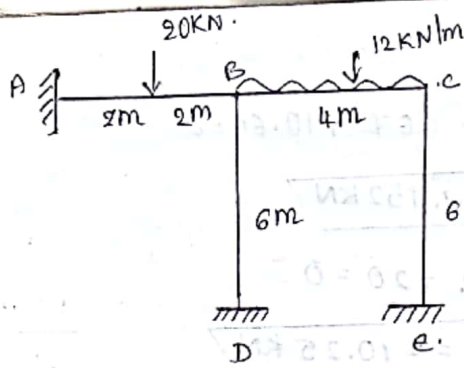
$$H_B - 9.752 + 20 = 0$$

$$H_B = -10.25 \text{ kN}$$

25



3



solⁿ: step 1: Fixed end moment.

$$\text{Span AB: } M_{FAB} = \frac{-wL}{8} = \frac{-20 \times 4}{8} \Rightarrow M_{FAB} = -10 \text{ kN-m}$$

$$M_{FBA} = \frac{wL}{8} = \frac{20 \times 4}{8} \Rightarrow M_{FBA} = 10 \text{ kN-m}$$

$$\text{Span BC: } M_{FBC} = \frac{-wL^2}{12} = \frac{-12 \times (4)^2}{12} \Rightarrow M_{FBC} = -16 \text{ kN-m}$$

$$M_{FCB} = \frac{wL^2}{12} = \frac{12 \times (4)^2}{12} \Rightarrow M_{FCB} = 16 \text{ kN-m}$$

$$M_{FBD} = M_{FDB} = 0 ; \quad M_{FCE} = M_{FEC} = 0$$

step 2: Slope-deflection equation. ; $\theta_A = 0$; $\theta_D = 0$; $\theta_E = 0$, $\delta = 0$

$$M_{AB} = M_{FAB} + \frac{2EI}{L} \left[2\theta_A + \theta_B - \frac{3\delta}{L} \right]$$

$$= -10 + \frac{2EI}{4} [\theta_B]$$

$$= -10 + 0.5EI\theta_B \rightarrow (1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L} \left[2\theta_B + \theta_A - \frac{3\delta}{L} \right]$$

$$= 10 + EI\theta_B \rightarrow (2)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L} \left[2\theta_B + \theta_C - \frac{3\delta}{L} \right]$$

$$= -16 + 0.5EI [2\theta_B + \theta_C]$$

$$= -16 + EI\theta_B + 0.5EI\theta_C \rightarrow (3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L} \left[2\theta_C + \theta_B - \frac{3\delta}{L} \right]$$

$$= 16 + 0.5EI\theta_B + EI\theta_C \rightarrow (4)$$

$$M_{BD} = M_{FBD} + \frac{2EI}{L} \left[2\theta_B + \theta_D - \frac{3\delta}{L} \right]$$

$$= 0 + 0.333 \times 0.66 EI\theta_B \rightarrow (5)$$

$$M_{DB} = M_{FDB} + \frac{2EI}{L} \left[2\theta_D + \theta_B - \frac{3\delta}{L} \right]$$

$$= 0 + 0.33 EI\theta_B \rightarrow (6)$$

$$M_{CE} = M_{FCE} + \frac{2EI}{L} \left[2\theta_C + \theta_E - \frac{3\delta}{L} \right]$$

$$= 0 + 0.666 EI\theta_C \rightarrow (7)$$

$$M_{EC} = M_{FEC} + \frac{2EI}{L} \left[2\theta_E + \theta_C - \frac{3\delta}{L} \right]$$

$$= 0 + 0.33EI\theta_C \rightarrow (8)$$

27

Step 3: Equilibrium Conditions.

$$M_{BA} + M_{BC} + M_{BD} = 0;$$

$$10 + EI\theta_B + 0.66EI\theta_B - 16 + EI\theta_B + 0.5EI\theta_C = 0.$$

$$2.66EI\theta_B + 0.5EI\theta_C = 6 \rightarrow (9).$$

$$M_{CB} + M_{CE} = 0;$$

$$16 + 0.5EI\theta_B + EI\theta_C + 0.666EI\theta_C = 0.$$

$$0.5EI\theta_B + 1.66EI\theta_C = -16 \rightarrow (10).$$

$$\theta_B = \frac{4.311}{EI}; \quad \theta_C = \frac{-10.9}{EI}$$

$$M_{AB} = -10 + 0.5EI \left[\frac{4.311}{EI} \right] \Rightarrow -7.844 \text{ KN-m}$$

$$M_{BA} = 10 + EI \left[\frac{4.311}{EI} \right] \Rightarrow 14.311 \text{ KN-m}$$

$$M_{BC} = -16 + EI \left[\frac{4.311}{EI} \right] + 0.5 \left[\frac{-10.9}{EI} \right] EI = -17.18 \text{ KN-m}$$

$$M_{CB} = 16 + 0.5EI \left[\frac{4.311}{EI} \right] + \left(\frac{10.9}{EI} \right) EI = 7.22 \text{ KN-m}$$

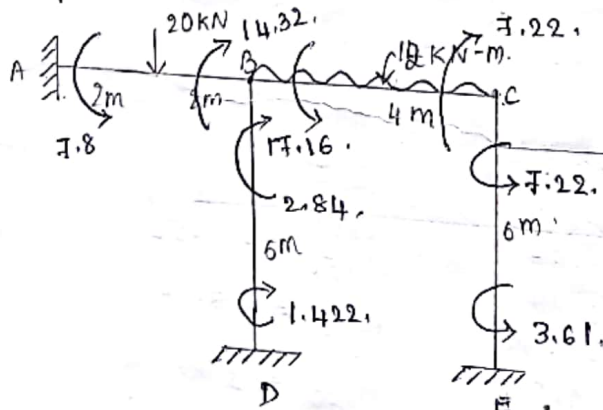
$$M_{BD} = 0.66EI \left[\frac{4.311}{EI} \right] = 2.84 \text{ KN-m}$$

$$M_{DB} = 0.33EI \left[\frac{4.311}{EI} \right] = 1.422 \text{ KN-m}$$

$$M_{CE} = 0.66EI \left[\frac{-10.9}{EI} \right] = -7.192 \text{ KN-m}$$

$$M_{EC} = 0.33EI \left[\frac{-10.9}{EI} \right] = -3.597 \text{ KN-m}$$

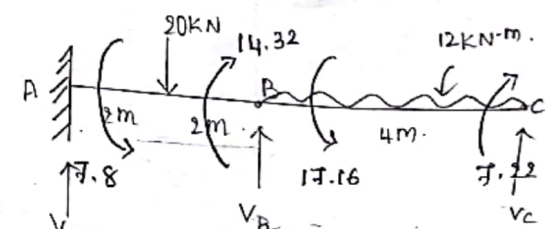
Step 4: SFD.



$$\sum M_B = 0;$$

$$-4V_C + 12 \times 4 \times \frac{4}{2} + 7.22 - 17.16 = 0.$$

$$V_C = 21.515 \text{ kN}$$



$$\text{Span AB: } \sum M_A = 0;$$

$$-4V_B + 14.32 + 20 \times 2 - 7.8 = 0$$

$$V_B = 11.63 \text{ kN}$$

$$\sum V = 0; \quad V_A + 11.63 - 20 = 0.$$

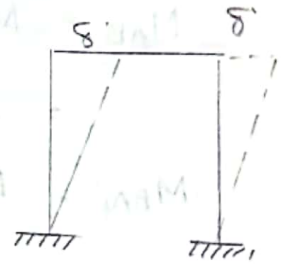
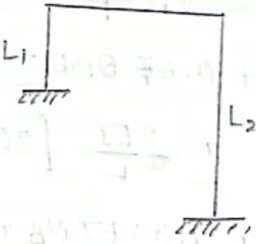
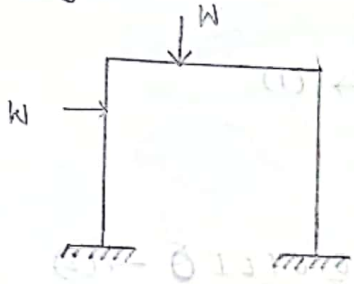
$$V_A = 8.37 \text{ kN}$$

Sway - Frames

The side movement of the end of the column in a frame is called "sway"

Unsymmetrical Loading

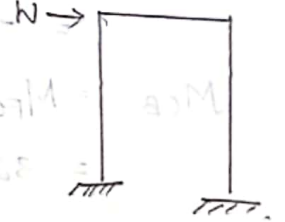
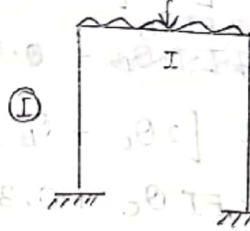
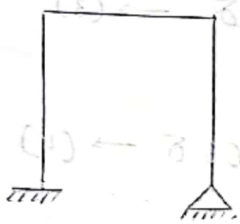
Unsymmetrical Shape.



Different end conditions

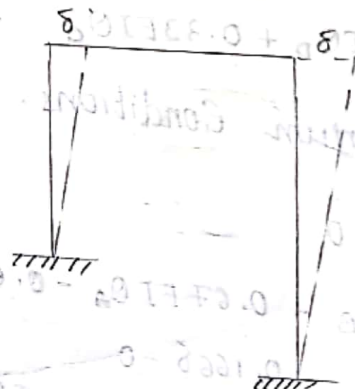
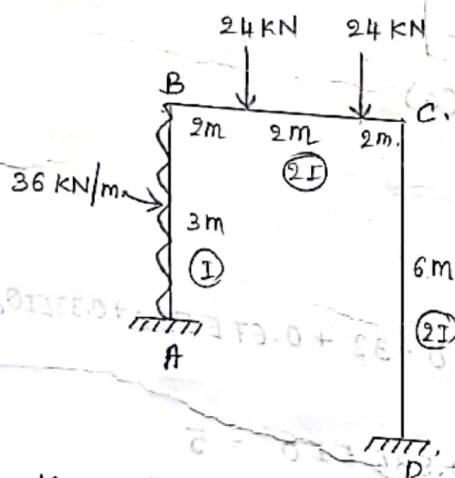
Non-Uniform Sections.

Horizontal Load.



Problems

1)



solⁿ: step 1: Fixed End Moments

Span AB, $M_{FAB} = \frac{-WL^2}{12} = \frac{-36(3)^2}{12} = -27 \text{ KN-m}$

$M_{FBA} = \frac{+WL^2}{12} = \frac{36(3)^2}{12} = 27 \text{ KN-m}$

Span BC, $M_{FBC} = \frac{-WD^2b^2}{L^2} - \frac{wa^2b^2}{L^2} = \frac{-24(2)^2(4)^2}{(6)^2} - \frac{24(2)(2)^2}{(6)^2}$

$M_{FBC} = -32 \text{ KN-m}$ $\begin{matrix} a=2 \\ b=4 \end{matrix}$ $\begin{matrix} a=4 \\ b=2 \end{matrix}$

$M_{FCB} = \frac{wa^2b}{L^2} + \frac{wa^2b}{L^2} = \frac{24(2)^2(4)}{6^2} + \frac{24(4)^2(2)}{6^2}$

$M_{FCB} = 32 \text{ KN-m}$

$$M_{FCD} = M_{FDC} = 0$$

(31)

STEP 2: Slope-Deflection equation

$$\theta_A = \theta_D = 0 \quad \text{For span AB \& DC, } \delta \neq 0$$

$$\text{BC, } \delta = 0$$

$$M_{AB} = M_{FAB} + \frac{2EI}{L} \left[2\theta_A + \theta_B - \frac{3\delta}{L} \right]$$

$$= -27 + 0.67EI\theta_B - 0.67EI\delta \rightarrow (1) \quad \text{AB}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L} \left[2\theta_B + \theta_A - \frac{3\delta}{L} \right]$$

$$= 27 + 1.33EI\theta_B + 0.67EI\theta_A - 0.67EI\delta \rightarrow (2) \quad \text{BA}$$

$$M_{BC} = M_{FBC} + \frac{2EI \times 2}{L} \left[2\theta_B + \theta_C - \frac{3\delta}{L} \right]$$

$$= -32 + 0.66EI\theta_B + 1.33EI\theta_C + 0.66EI\delta \rightarrow (3) \quad \text{BC}$$

$$M_{CB} = M_{FCB} + \frac{4EI}{L} \left[2\theta_C + \theta_B - \frac{3\delta}{L} \right]$$

$$= 32 + 1.33EI\theta_C + 0.66EI\theta_B \rightarrow (4) \quad \text{CB}$$

$$M_{CD} = M_{FCD} + \frac{4EI}{L} \left[2\theta_C + \theta_D - \frac{3\delta}{L} \right]$$

$$= 1.33EI\theta_C - 0.33EI\delta \rightarrow (5) \quad \text{CD}$$

$$M_{DC} = M_{FDC} + \frac{4EI}{L} \left[2\theta_D + \theta_C - \frac{3\delta}{L} \right]$$

$$= 0.67EI\theta_C - 0.33EI\delta \rightarrow (6) \quad \text{DC}$$

step 3: Equilibrium condition.

$$M_{BC} + M_{BA} = 0$$

$$27 + 1.33EI\theta_B - 0.67EI\delta - 32 + 1.33EI\theta_B + 0.66EI\theta_C = 0$$

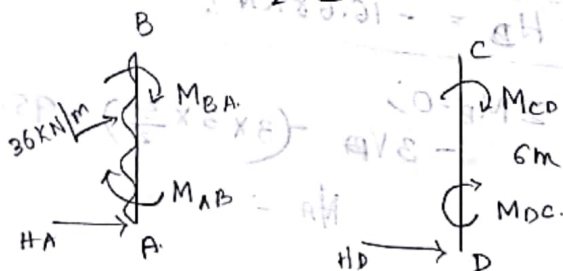
$$2.66EI\theta_B + 0.66EI\theta_C - 0.67EI\delta = 5 \rightarrow (7)$$

$$M_{CB} + M_{CD} = 0$$

$$32 + 1.33EI\theta_C + 0.66EI\theta_B + 1.33EI\theta_C - 0.33EI\delta$$

$$0.66EI\theta_B + 2.66EI\theta_C - 0.33EI\delta = -32 \rightarrow (8)$$

Considering free body diagrams of columns & taking moment about top joints we get.



$$\Sigma M = 0; \quad H_A + H_D + (36 \times 3) = 0. \quad \rightarrow *$$

$$\Sigma M_B = 0; \quad -3H_A - 36 \times 3 \times \frac{3}{2} + M_{AB} + M_{BA} = 0$$

$$H_A = \frac{M_{AB} + M_{BA} - 162}{3}$$

$$\Sigma M_C = 0; \quad -6H_D + M_{DC} + M_{CD} = 0.$$

$$H_D = \frac{M_{DC} + M_{CD}}{6}$$

Sub H_A & H_D in * eqⁿ.

$$\frac{M_{AB} + M_{BA} - 162}{3} + \frac{M_{DC} + M_{CD}}{6} + 108 = 0.$$

$$2M_{AB} + 2M_{BA} - 324 + M_{DC} + M_{CD} + 216 = 0$$

$$2M_{AB} + 2M_{BA} + M_{DC} + M_{CD} = -102 - 216$$

$$2M_{AB} + 2M_{BA} + M_{DC} + M_{CD} = -324$$

$$2(-27 + 0.67EI\theta_A - 0.67EI\delta) + 2(27 + 1.33EI\theta_B - 0.67EI\delta) + 0.67EI\theta_C - 0.33EI\delta + 1.33EI\theta_C - 0.33EI\delta = -324,$$

$$-54 + 1.34EI\theta_B - 1.2EI\delta + 54 + 2.66EI\theta_B - 1.34EI\delta + 0.67EI\theta_C - 0.33EI\delta + 1.33EI\theta_C - 0.33EI\delta = -324.$$

$$4EI\theta_B + 2EI\theta_C - 3.24EI\delta = -324.$$

$$\theta_B = \frac{38.5}{EI}; \quad \theta_C = \frac{-4.1}{EI}; \quad \delta = \frac{140.3}{EI}.$$

$$M_{AB} = -95.3 \text{ KN-m}$$

$$M_{BA} = -16.2 \text{ KN-m}$$

$$M_{BC} = 15.9 \text{ KN-m}$$

$$M_{DC} = -48.8 \text{ KN-m}$$

$$M_{CD} = 51.3 \text{ KN-m}$$

$$M_{CP} = 51.3 \text{ KN-m}$$

STIFFNESS MATRIX

The analysis is carried out by using $\{P\} = [K] \{D\}$ or

$\{D\} = -[K]^{-1} \{P\}$

In general, $\{P\} + [K] \{D\} = 0$

$\{D\} = -[K]^{-1} \{P\}$

$$\begin{Bmatrix} D_1 \\ D_2 \end{Bmatrix} = - \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}^{-1} \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix}$$

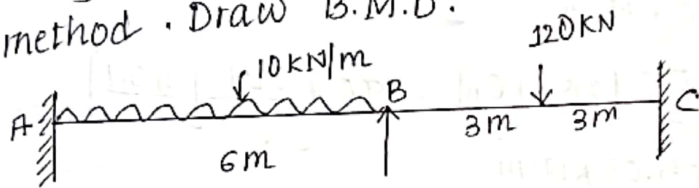
$$\begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \end{Bmatrix} = 0$$

$P_1 + K_{11} D_1 + K_{12} D_2 = 0 \rightarrow (1)$

$P_2 + K_{21} D_1 + K_{22} D_2 = 0 \rightarrow (2)$

Problems

1] Analyse the continuous beam ABC shown in fig by stiffness method. Draw B.M.D.



EI = constant

The ends of the continuous beam ABC is fixed hence the slopes at fixed supports are 0

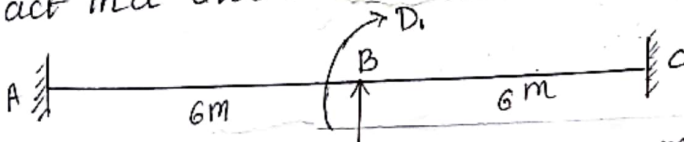
Thus, $\theta_A = \theta_C = 0$

step 1: Fixed end Method.

$M_{FAB} = \frac{-wL^2}{12} = \frac{-10 \times (6)^2}{12} \Rightarrow M_{FAB} = -30 \text{ kN}$

The support B is an intermediate support & $\theta_B \neq 0$. Let this kinematic redundant is denoted as D_1 . This is assumed to act in a clockwise direction.

$\Rightarrow D_1 = \theta_B$



$M_{FBA} = \frac{+wL^2}{12} = \frac{10 \times (6)^2}{12} = 30 \text{ kN-m}$

$M_{FBC} = \frac{-wL}{8} = \frac{-120 \times (6)}{8} = -90 \text{ kN-m}$

$M_{FCB} = \frac{+wL}{8} = \frac{120 \times 6}{8} = 90 \text{ kN-m}$

Step 2: Joint load.

$P = M_{FBA} + M_{FBC} = +30 - 90 = -60 \text{ kN-m}$

K_{11} = Unit rotation @ B creates a moment @ B.

step 3: stiffness coefficients or stiffness matrix.

$$K_{11} = \left(\frac{4EI}{L}\right)_{BA} + \left(\frac{4EI}{L}\right)_{BC} = \frac{4EI}{6} + \frac{4EI}{6} = 1.33EI$$

step 4: Joint Equilibrium.

$$P_1 + K_{11} D_1 = 0$$

$$-60 + 1.33EI D_1 = 0$$

$$\theta_B = D_1 = \frac{45.11}{EI}$$

Assume all the supports are fixed.

The end moments are calculated by using slope-deflection equations.

$$M_{AB} = M_{FAB} + \frac{2EI}{L} [2\theta_A + \theta_B] = -30 + \frac{2EI}{6} \left[\frac{45.1}{EI} \right]$$

$$M_{AB} = -14.967 \approx -15$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L} [2\theta_B + \theta_A] = 30 + \frac{2EI}{6} \left[\frac{2(45.1)}{EI} \right]$$

$$M_{BA} = 60.067 \text{ KN-m}$$

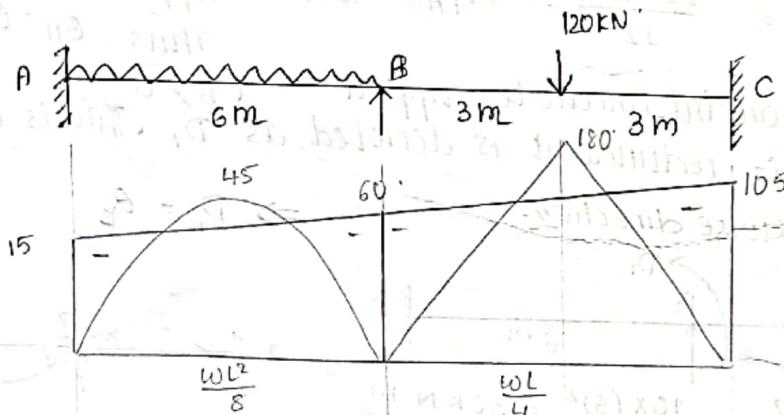
$$M_{BC} = M_{FBC} + \frac{2EI}{L} [2\theta_B + \theta_C] = -90 + \frac{2EI}{6} \left[\frac{2(45.1)}{EI} \right]$$

$$M_{BC} = -60 \text{ KN-m}$$

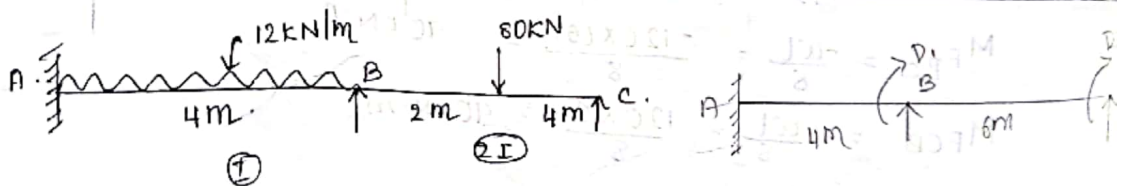
$$M_{CB} = M_{FCB} + \frac{2EI}{L} [2\theta_C + \theta_B] = 90 + \frac{2EI}{6} \left[\frac{45.1}{EI} \right]$$

$$M_{CB} = 105.03 \text{ KN-m}$$

Draw B.M.D



9]



step 1: Fixed end Moment

$$M_{FAB} = \frac{-wL^2}{12} = \frac{-12(4)^2}{12} = -16 \text{ KN-m}$$

$$M_{FBA} = \frac{wL^2}{12} = \frac{12(4)^2}{12} = 16 \text{ KN-m}$$

$$M_{FBC} = -\frac{w a b^2}{L^2} = -\frac{80 \times 2(4)^2}{(6)^2} = -71.11 \text{ KN-m}$$

$$M_{FCB} = \frac{w a^2 b}{L^2} = \frac{80 \times (2)^2 \times 4}{(6)^2} = 35.56 \text{ KN-m}$$

step 2: Joint load.

$$P_1 = M_{FBA} + M_{FBC}$$

$$= 16 - 71.11$$

$$P_1 = -55.11$$

$$P_2 = M_{FCB}$$

$$P_2 = 35.56 \text{ KN-m}$$

step 3: Stiffness coefficient. [Applying unit moment at B]

$$K_{11} = \left(\frac{4EI}{L}\right)_{BA} + \left(\frac{4EI}{L}\right)_{BC} = \frac{4EI}{4} + \frac{4EI \times 2}{6} = 2.33EI$$

$$K_{12} = \left(\frac{2EI}{L}\right)_{BC} = 0.67EI$$

$$K_{21} = \left(\frac{2EI}{L}\right)_{BC} = \frac{2 \times 2EI}{6} = 0.67EI$$

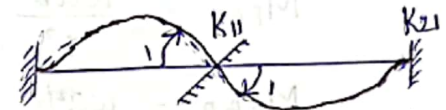
$$K_{22} = \left(\frac{4EI}{L}\right)_{BC} = \frac{4 \times 2EI}{6} = 1.33EI$$

K_{11} = moment required at B due to unit rotation of B at B

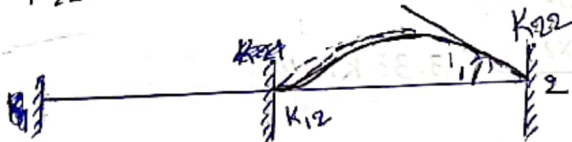
K_{21} = moment required at B due to develop at C due to unit rotation at B.

K_{12} = moment caused at B due to unit rotation at C

K_{22} = moment required at C due to unit rotation at C



Assume all the supports are fixed



$$[K] = \begin{bmatrix} 2.33 & 0.67 \\ 0.67 & 1.33 \end{bmatrix} EI$$

step 4: Joint Equilibrium

$$P_1 + K_{11} D_1 + K_{12} D_2 = 0 \Rightarrow -55.11 = 2.33 D_1 + 0.67 D_2$$

$$P_2 + K_{21} D_1 + K_{22} D_2 = 0 \Rightarrow -35.56 = 0.67 D_1 + 1.33 D_2$$

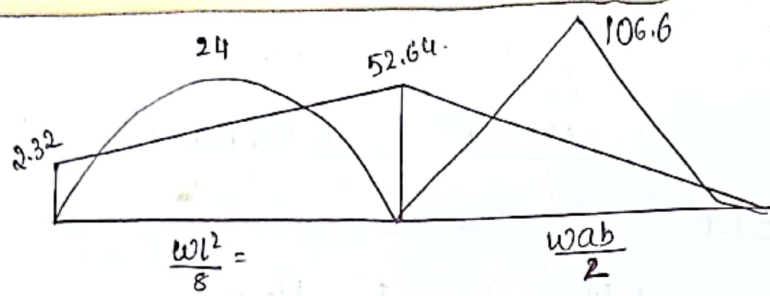
$$D_B = D_1 = \frac{36.64}{EI} \text{ KN-m}, D_C = D_2 = \frac{-45.19}{EI} \text{ KN-m}$$

$$M_{AB} = M_{FAB} + \frac{2EI}{L} [2D_A + D_B] = -16 + \frac{2EI}{4} [36.64] = 2.32$$

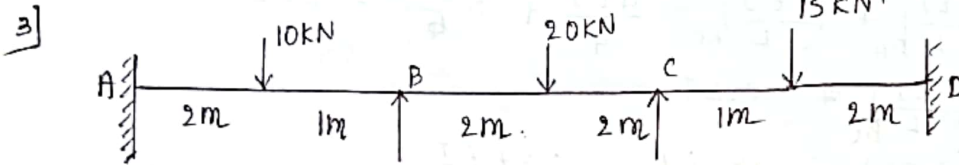
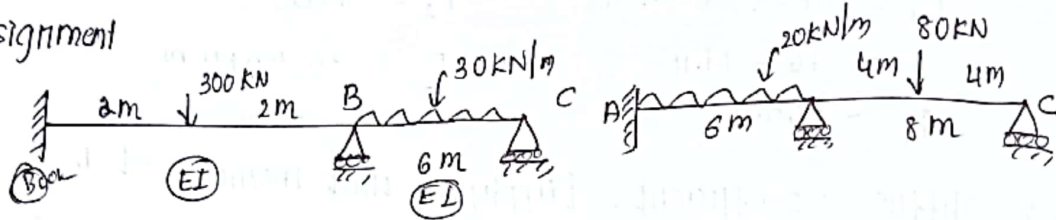
$$M_{BA} = M_{FBA} + \frac{2EI}{L} [2D_B + D_C] = 16 + \frac{2EI}{4} [2(36.64)] = 52.64$$

$$M_{BC} = M_{FBC} + \frac{4EI}{L} [2D_B + D_C] = -71.11 + \frac{4EI}{6} [2(36.64) - 45.19] = -52.383$$

$$M_{CB} = M_{FCB} + \frac{4EI}{L} [2D_C + D_B] = 35.56 + \frac{4EI}{6} [2(-45.19) + 36.64] = 0$$



Assignment



Step 1: Fixed End Moment

$$M_{FAB} = -\frac{wab^2}{L^2} = -\frac{10 \times 2 \times (1)^2}{(3)^2} = -2.22 \text{ kN}\cdot\text{m}$$

$$M_{FBA} = \frac{wa^2b}{L^2} = \frac{10 \times (2)^2 \times 1}{(3)^2} = 4.44 \text{ kN}\cdot\text{m}$$

$$M_{FBC} = -\frac{wL^2}{8} = -\frac{20 \times (4)}{8} = -10 \text{ kN}\cdot\text{m}$$

$$M_{FCB} = \frac{wL}{8} = \frac{20 \times (4)}{8} = 10 \text{ kN}\cdot\text{m}$$

$$M_{FCD} = -\frac{wab^2}{L^2} = -\frac{15 \times (1)^2 \times 2^2}{(3)^2} = -6.67 \text{ kN}\cdot\text{m}$$

$$M_{FDC} = \frac{wa^2b}{L^2} = \frac{15 \times (1)^2 \times 2}{(3)^2} = 3.33 \text{ kN}\cdot\text{m}$$

Step 2: Joint Stiffness

$$P_1 = M_{FBA} + M_{FBC} = 2.22 - 10 = -7.78 \text{ kN}\cdot\text{m}$$

$$P_2 = M_{FCB} + M_{FCD} = 10 - 6.67 = 3.33 \text{ kN}\cdot\text{m}$$

Step 3: Stiffness Co-efficient

$$K_{11} = \left[\frac{4EI}{L} \right]_{BA} + \left[\frac{4EI}{L} \right]_{BC} = \left[\frac{4EI}{3} \right] + \left[\frac{4EI}{4} \right] = 2.33EI$$

$$K_{12} = \left[\frac{2EI}{L} \right]_{BC} = \frac{2EI}{4} = 0.5EI$$

$$K_{21} = \left[\frac{2EI}{L} \right]_{CB} = \frac{2EI}{4} = 0.5EI$$

$$K_{22} = \left[\frac{4EI}{L} \right]_{CB} + \left[\frac{4EI}{L} \right]_{CD} = 2.33EI$$

$$P_1 + K_{11} D_1 + K_{12} D_2 = 0$$

$$5.55 = 2.33 D_1 + 0.5 D_2$$

$$P_2 + K_{21} D_1 + K_{22} D_2 = 0$$

$$-3.33 = 0.5 D_1 + 2.33 D_2$$

$$D_1 = \theta_B = \frac{2.81}{EI} \text{ KN-m}$$

$$D_2 = \theta_C = \frac{-2.03}{EI} \text{ KN-m}$$

$$M_{AB} = M_{FAB} + \frac{2EI}{L} [2\theta_A + \theta_B] = -2.22 + \frac{2EI}{3} \left[\frac{2.81}{EI} \right] = -0.3467$$

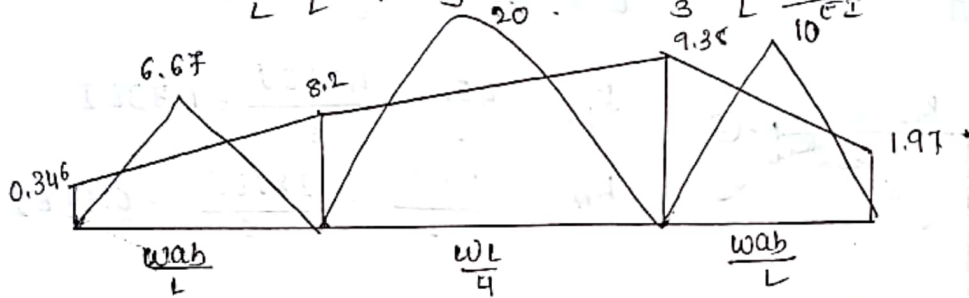
$$M_{BA} = M_{FBA} + \frac{2EI}{L} [2\theta_B + \theta_A] = 4.44 + \frac{2EI}{3} \left[\frac{2(2.81)}{EI} \right] = 8.186$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L} [2\theta_B + \theta_C] = -10 + \frac{2EI}{4} \left[\frac{2(2.81) + (-2.03)}{EI} \right] = -8.205$$

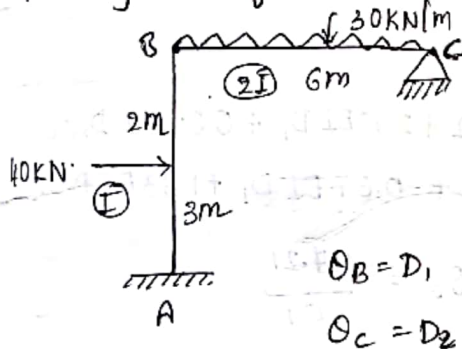
$$M_{CB} = M_{FCB} + \frac{2EI}{L} [2\theta_C + \theta_B] = 10 + \frac{2EI}{4} \left[\frac{2(-2.03) + 2.81}{EI} \right] = 9.38$$

$$M_{CD} = M_{FCD} + \frac{2EI}{L} [2\theta_C + \theta_D] = -10 - 6.667 + \frac{2EI}{4} \left[\frac{2(-2.03)}{EI} \right] = -9.38$$

$$M_{DC} = M_{FDC} + \frac{2EI}{L} [2\theta_D + \theta_C] = 3.33 + \frac{2EI}{3} \left[\frac{-2.03}{EI} \right] = 1.97$$

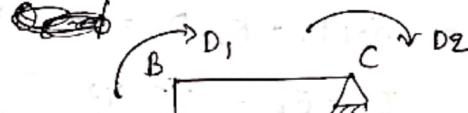


Analysis of Frames



$$\theta_A = 0 \quad [\because \text{Fixed Support}]$$

$$\theta_B \text{ \& } \theta_C \text{ are redundants \& } \theta_B \neq 0, \theta_C \neq 0$$



$$\theta_B = D_1$$

$$\theta_C = D_2$$

Step 1: F.E.M

$$M_{FAB} = -\frac{wab^2}{L^2} = \frac{-40 \times 3(2)^2}{(5)^2} = -19.2 \text{ KN-m}$$

$$M_{FBA} = \frac{wa^2b}{L^2} = \frac{40 \times (3)^2 \times 2}{(5)^2} = 28.8 \text{ KN-m}$$

$$M_{FBC} = -\frac{wl^2}{12} = \frac{-30 \times (6)^2}{12} = -90 \text{ KN-m}$$

$$M_{FCB} = \frac{wl^2}{12} = \frac{30 \times (6)^2}{12} = 90 \text{ KN-m}$$

Step 2: Joint load.

$$P_1 = M_{FBA} + M_{FBC}$$

$$= 28.8 + (-90)$$

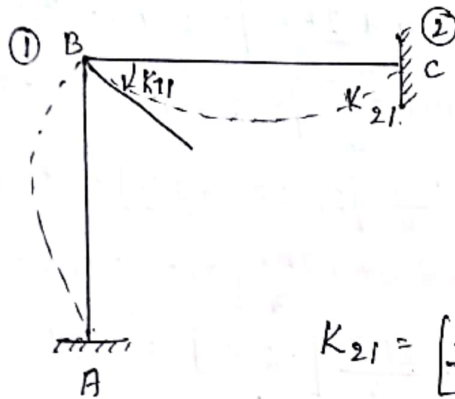
$$P_1 = -61.2 \text{ KN-m}$$

$$P_2 = M_{FCB} = 90 \text{ KN-m}$$

Step 3: Stiffness Co-efficient

Apply unit moment at B & C

at B



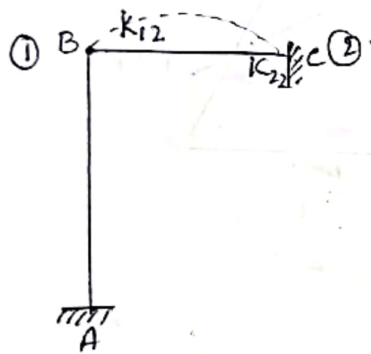
$$K_{11} = \left[\frac{4EI}{L} \right]_{BA} + \left[\frac{4EI}{L} \right]_{BC}$$

$$K_{11} = \frac{4 \times 2EI}{5} + \frac{4 \times 2EI}{6}$$

$$K_{11} = 2.13EI$$

$$K_{21} = \left[\frac{2EI}{L} \right]_{CB} = \frac{2 \times 2EI}{6} = 0.67EI$$

at C



$$K_{22} = \frac{4EI}{L} = \frac{4 \times 2EI}{6} = 1.33EI$$

$$K_{12} = \frac{2EI}{L} = \frac{2 \times 2EI}{6} = 0.67EI$$

STEP 4: Joint Equilibrium.

$$P_1 + K_{11}D_1 + K_{12}D_2 = 0 \Rightarrow -61.2 + 2.13EI D_1 + 0.67EI D_2 = 0$$

$$P_2 + K_{21}D_1 + K_{22}D_2 = 0 \Rightarrow 90 + 0.67EI D_1 + 1.33EI D_2 = 0$$

$$D_1 = \theta_B = \frac{59.43}{EI}, \quad D_2 = \theta_C = \frac{-97.21}{EI}$$

STEP 5: End Moments

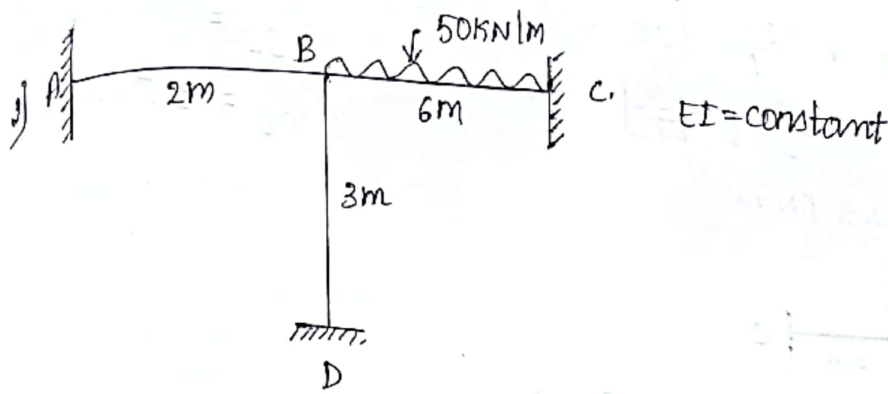
$$M_{AB} = M_{FAB} + \frac{2EI}{L} \left[2\theta_A + \theta_B - \frac{3\delta}{L} \right] = -19.2 + \frac{2EI}{5} \left[\frac{59.43}{EI} \right]$$

$$M_{AB} = -4.57 \text{ KN-m}$$

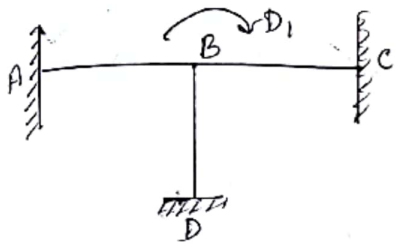
$$M_{BA} = 28.8 + \frac{2EI}{5} \left[2 \times \frac{59.43}{EI} \right] = 76.34 \text{ KN-m}$$

$$M_{BC} = -90 + \frac{2 \times 2EI}{6} \left[2 \times \frac{59.43}{EI} - \frac{97.61}{EI} \right] = -76 \text{ KN-m}$$

$$M_{CB} = 90 + \frac{2 \times 2EI}{6} \left[2 \times \frac{-97.61}{EI} + \frac{59.43}{EI} \right] = 0$$



$\theta_A = \theta_D = \theta_C = 0$, θ_B is the redundant hence $\theta_B \neq 0$
 $\Rightarrow \theta_B = D_1$



STEP 1: F.E.M

$$M_{FAB} = M_{FBA} = 0$$

$$M_{FDB} = M_{FBD} = 0$$

$$M_{FBC} = \frac{-wL^2}{12} = \frac{-50 \times (6)^2}{12} = -150 \text{ kN-m}$$

$$M_{FCB} = \frac{wL^2}{12} = \frac{50 \times (6)^2}{12} = 150 \text{ kN-m}$$

STEP 2: Joint load.

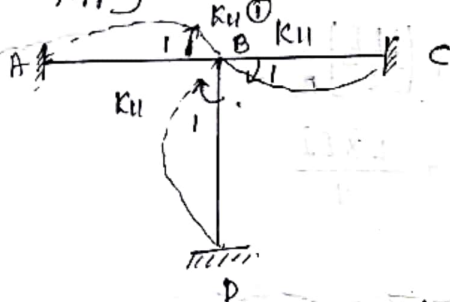
$$P_1 = M_{FBA} + M_{FBD} + M_{FBC}$$

$$P_1 = 0 + 0 - 150$$

$$P_1 = -150 \text{ kN-m}$$

STEP 3: Stiffness co-efficient

Apply unit moment @ B



$$K_{11} = \left[\frac{4EI}{L} \right]_{BA} + \left[\frac{4EI}{L} \right]_{BC} + \left[\frac{4EI}{L} \right]_{BD}$$

$$K_{11} = \frac{4EI}{2} + \frac{4EI}{6} + \frac{4EI}{3}$$

$$K_{11} = 4EI$$

STEP 4: Joint Equilibrium

$$P_1 + K_{11} D_1 = 0$$

$$-150 + 4EI D_1 = 0$$

$$\theta_B = D_1 = \frac{150}{4EI} = \frac{37.5}{EI}$$

STEP 5: End Moment

$$M_{BC} = M_{FBC} + \frac{2EI}{L} \left[2\theta_B + \theta_C - \frac{3\delta}{L} \right]$$

$$= -150 + \frac{2EI}{6} \left[2 \times \frac{37.5}{EI} \right]$$

$$M_{BC} = -125 \text{ kN-m}$$

$$M_{AB} = 37.5 \text{ kN-m}$$

$$M_{BA} = 75 \text{ kN-m}$$

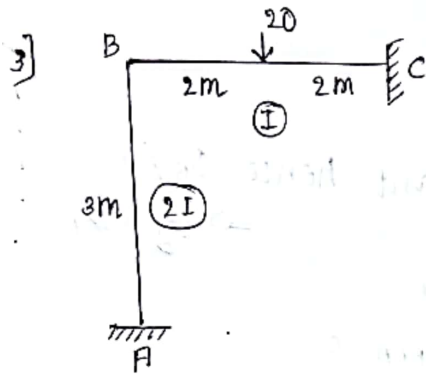
$$M_{CB} = M_{FCB} + \frac{2EI}{L} \left[2\theta_C + \theta_B - \frac{3\delta}{L} \right]$$

$$= 150 + \frac{2EI}{6} \left[\frac{37.5}{EI} \right]$$

$$M_{CB} = 162.5 \text{ KN}\cdot\text{m}$$

$$M_{BD} = 50 \text{ KN}\cdot\text{m}$$

$$M_{DB} = 25 \text{ KN}\cdot\text{m}$$



$\theta_A = \theta_C = 0$;
 θ_B are redundant, $\theta_B \neq 0$

STEP 1: F.E.M.

$$M_{FAB} = M_{FBA} = 0$$

$$M_{FBC} = -\frac{wL}{8} = -\frac{20 \times 4}{8} = -10 \text{ KN}\cdot\text{m}$$

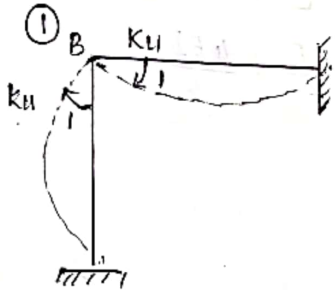
$$M_{FCB} = \frac{wL}{8} = \frac{20 \times 4}{8} = 10 \text{ KN}\cdot\text{m}$$

STEP 2: Joint load.

$$P_i = M_{FBA} + M_{FBC} = 0 - 10 = -10 \text{ KN}\cdot\text{m}$$

STEP 3: Stiffness Co-efficient

Apply unit moment at B.



$$K_{11} = \left[\frac{4EI}{L} \right]_{BA} + \left[\frac{4EI}{L} \right]_{BC}$$

$$= \frac{4 \times 2EI}{3} + \frac{4 \times EI}{4}$$

$$K_{11} = 3.67 EI$$

STEP 4: Joint Equilibrium.

$$P_i + K_{11} D_i = 0$$

$$-10 + 3.67 EI \times D_i = 0$$

$$\theta_B = D_i = \frac{2.72}{EI}$$

STEP 5: End Moment

$$M_{BC} = M_{FBC} + \frac{2EI}{L} \left[2\theta_B + \theta_C - \frac{3\delta}{L} \right]$$

$$= -10 + \frac{2EI}{L} \left[2 \times \frac{2.72}{EI} \right]$$

$$M_{BC} = -7.28 \text{ KN}\cdot\text{m}$$

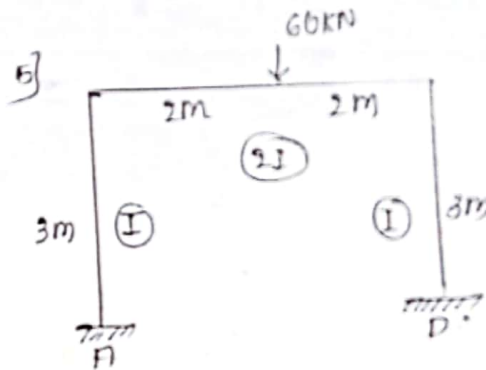
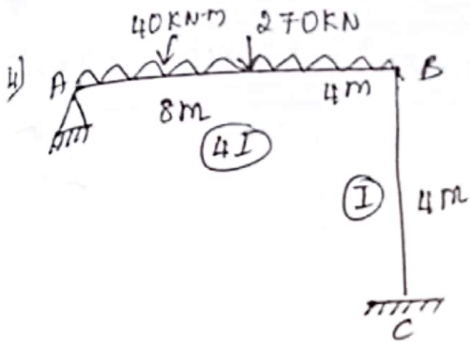
$$M_{AB} = 3.64 \text{ KN}\cdot\text{m}$$

$$M_{BA} = 7.28 \text{ KN}\cdot\text{m}$$

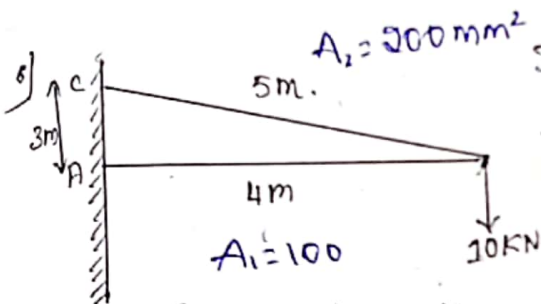
$$M_{CB} = M_{FCB} + \frac{2EI}{L} \left[2\theta_C + \theta_B - \frac{3\delta}{L} \right]$$

$$= 10 + \frac{2EI}{4} \left[\frac{2.72}{EI} \right]$$

$$M_{CB} = 11.36 \text{ kN}$$



Analysis of Pin Jointed Trusses



Determine the displacement at the free end & forces in all the members of the pin joint plane frame.

shown in fig by stiffness matrix & area of e/s member.
 $E = 2 \times 10^5 \text{ N/mm}^2$

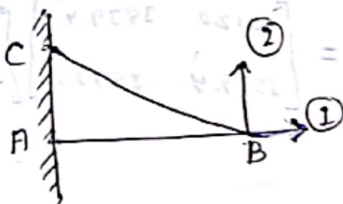
Soln. STEP 1: Kinematic Indeterminacy

$$DKI = 2J - r$$

$$= 2 \times 3 - 4$$

$$DKI = 2$$

STEP 2: Identify the co-ordinates

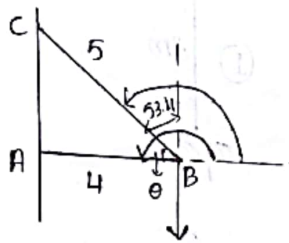


Applied forces at co-ordinates 1 & 2 are.

$$\begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -10 \end{Bmatrix} \text{ kN}$$

STEP 3: Develop a stiffness matrices, introducing a unit displacement at co-ordinate 1 & 2 successively & compute stiffness co-efficients.

Member	$\frac{AE}{L}$ KN/mm	θ degree	$\cos\theta$	$\sin\theta$	$\frac{AE}{L} \cos^2\theta$ KN/mm	$\frac{AE}{L} \sin^2\theta$ KN/mm	$\frac{AE}{L} \cos\theta \sin\theta$ KN/mm
AB	5×10^3	180	-1	0	5×10^3	0	0
BC	8×10^3	143.13	-0.8	0.6	5.12×10^3	2.88×10^3	-3.84×10^3
			Σ		10.12×10^3	2.86×10^3	-3.84×10^3



$$\cos\theta = \frac{4}{5}$$

$$\theta = 36.86$$

$$90 - 36.86 = 53.14$$

$$90 + 53.14 = 143.14$$

STEP 4: Stiffness Co-efficients.

$$K_{11} = \Sigma \frac{AE}{L} \cos^2\theta$$

$$K_{21} = K_{12} = \Sigma \frac{AE}{L} \cos\theta \sin\theta$$

$$K_{22} = \Sigma \frac{AE}{L} \sin^2\theta$$

$$K_{11} = 10.12 \times 10^3$$

$$K_{21} = K_{12} = -3.84 \times 10^3$$

$$K_{22} = 2.86 \times 10^3$$

stiffness matrix.

$$[K] = \begin{bmatrix} 10.12 & -3.84 \\ -3.84 & 2.86 \end{bmatrix} 10^3$$

step 5: Calculate the Displacement using the eqn

$$\{D\} = + [K]^{-1} \{P\}$$

$$\{P\} = [X] \{D\}$$

$$\begin{Bmatrix} D_1 \\ D_2 \end{Bmatrix} = 10^{-3} \begin{bmatrix} 10.12 & -3.84 \\ -3.84 & 2.86 \end{bmatrix}^{-1} \begin{Bmatrix} 0 \\ -10 \end{Bmatrix} = \begin{bmatrix} 10120 & 3839.4 \\ 3839.4 & 2879.2 \end{bmatrix} \begin{Bmatrix} 0 \\ -1000 \end{Bmatrix}$$

$$\begin{Bmatrix} D_1 \\ D_2 \end{Bmatrix} = \begin{Bmatrix} 2.67 \\ -7.02 \end{Bmatrix} \begin{matrix} \rightarrow D_{A2} \\ \text{mm} \\ \rightarrow D_{A1} \end{matrix}$$

Member forces can be calculated by using the formula

$$F_{AB} = \left(\frac{-AE}{L} \right)_{AB} \{ (D_{Ax} - D_{Bx}) \cos\theta_{AB} + (D_{Ay} - D_{By}) \sin\theta_{AB} \}$$

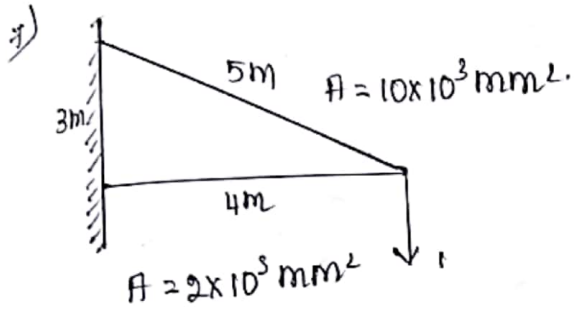
$$F_{AB} = F_1 = -5 \times 10^3 \{ (2.67 - 0) \cos 180^\circ + (-7.02 - 0) \sin 180^\circ \}$$

$$F_{AB} = 13.35 \times 10^3 \text{ N}$$

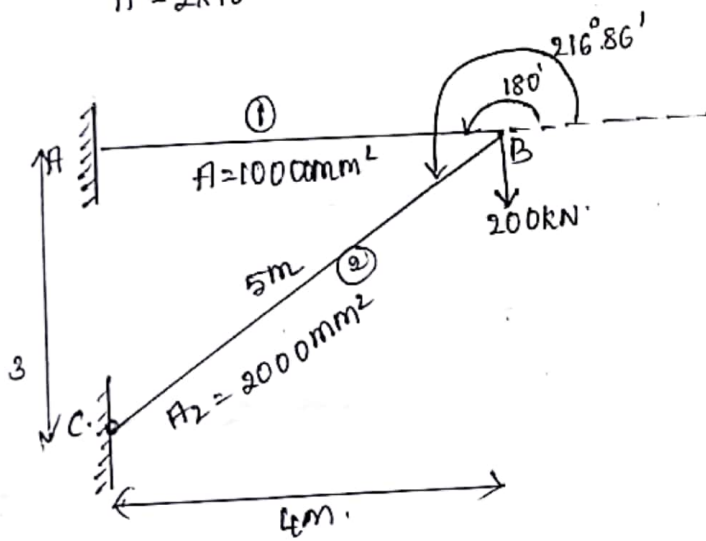
$$F_{AB} = 13.35 \text{ kN}$$

$$F_2 = F_2 = -8 \times 10^3 \left[(2.67 - 0) \cos 143.13 + (7.03 - 0) \sin 143.13 \right]$$

$$F_2 = 16.65 \text{ kN}$$



$$E = 2 \times 10^3 \text{ N/mm}^2$$



$$E = 20000 \text{ kN/mm}^2$$

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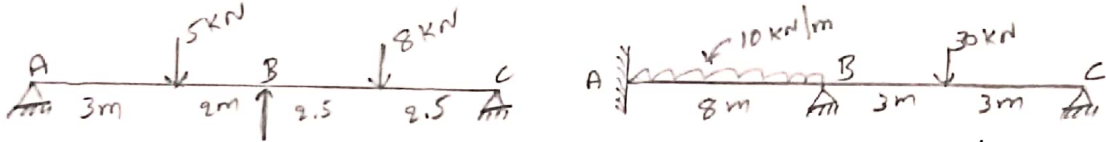
Department of Civil Engineering

ANALYSIS OF INDETERMINATE STRUCTURES (15CV52)

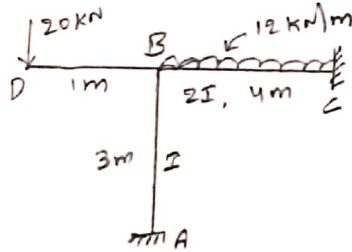
SEMESTER : V
 STAFF NAME : Gagan Krishna R R

ASSIGNMENT NUMBER : 1
 LAST DATE FOR SUBMISSION : 08.09.2018

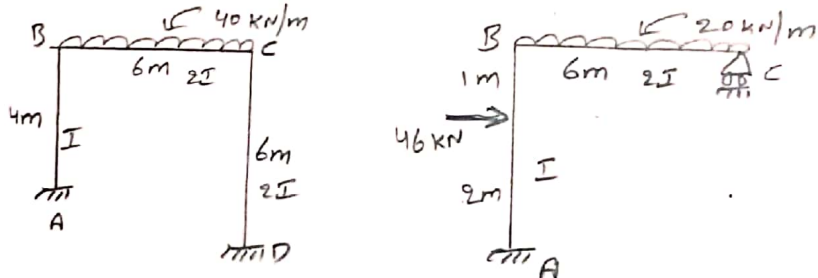
1. Analyze the continuous beam shown below by using Slope Deflection Method. Draw BMD. Take $EI = \text{Constant}$.



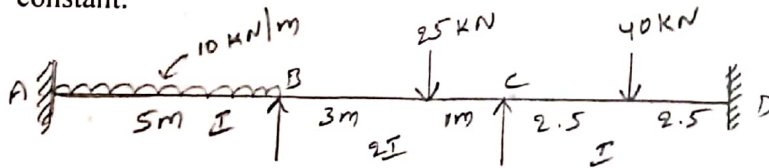
2. Analyze the frame shown below by using Slope Deflection Method. Draw BMD. Take $EI = \text{Constant}$.



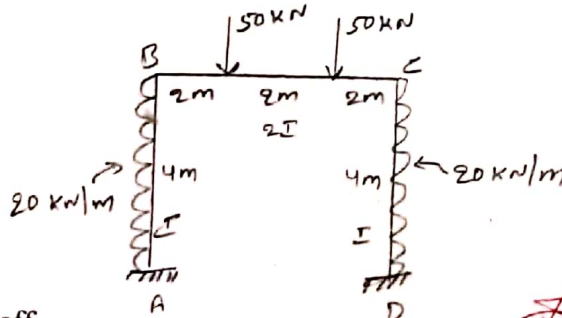
3. Analyze the frame shown below by using Slope Deflection Method. Draw BMD. Take $EI = \text{Constant}$.



4. Analyze the continuous beam shown below by using Moment Distribution Method. Draw BMD. Take $EI = \text{constant}$.



5. Analyze the frame shown below by using Moment Distribution Method. Draw BMD. Take $EI = \text{Constant}$.



Signature of Staff

Signature of HOD

Head, Dept of Civil Engg,
B G S Institute of Technology,
B G Nagara - 571448

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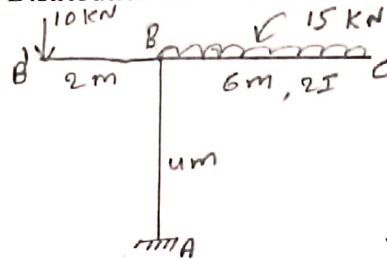
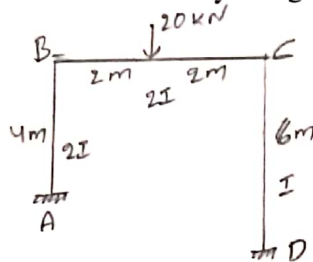
Department of Civil Engineering

ANALYSIS OF INDETERMINATE STRUCTURES (15CV52)

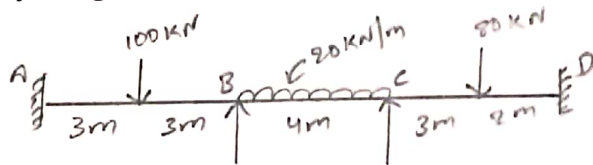
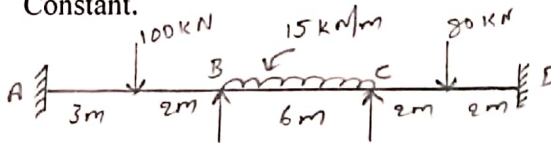
SEMESTER : V
STAFF NAME : Gagan Krishna R R

ASSIGNMENT NUMBER : 2
LAST DATE FOR SUBMISSION : 15.10.2018

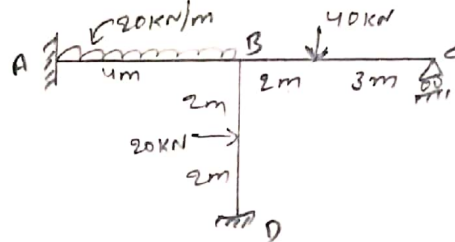
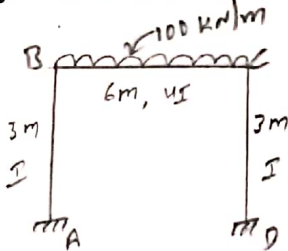
1. Analyze the frame shown below by using Moment Distribution Method. Draw BMD. Take $EI = \text{Constant}$.



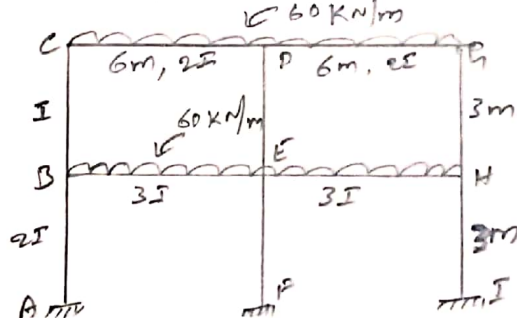
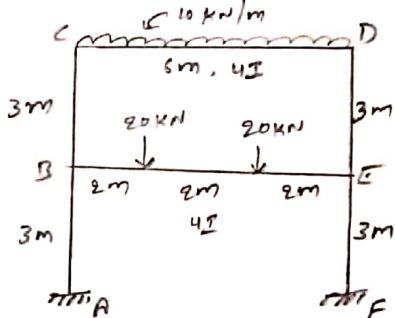
2. Analyze the continuous beam shown below by using Kani's Method. Draw BMD. Take $EI = \text{Constant}$.



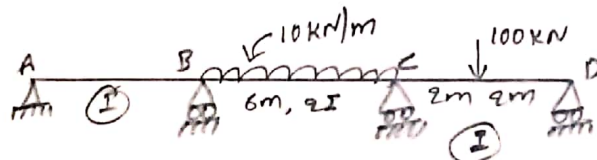
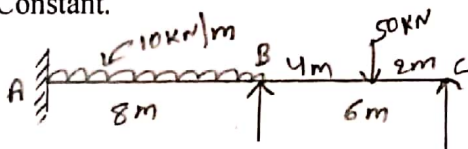
3. Analyze the frame shown below by using Kani's Method. Draw BMD. Take $EI = \text{Constant}$.

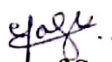



4. Analyze the frame shown below by using Kani's Method. Draw BMD. Take $EI = \text{constant}$.



5. Analyze the continuous beam shown below by using Flexibility Method. Draw BMD. Take $EI = \text{Constant}$.




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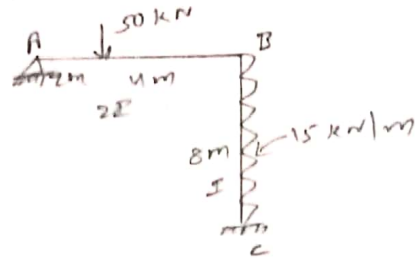
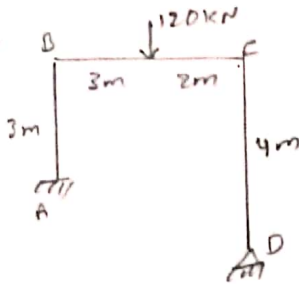
Department of Civil Engineering

ANALYSIS OF INDETERMINATE STRUCTURES (15CV52)

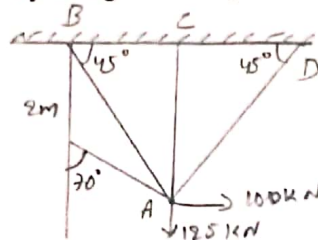
SEMESTER : V
STAFF NAME : Gagan Krishna R R

ASSIGNMENT NUMBER : 3
LAST DATE FOR SUBMISSION : 22.11.2018

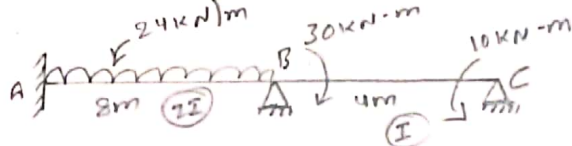
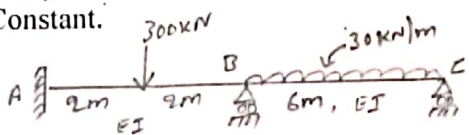
1. Analyze the frame shown below by using flexibility Method. Draw BMD. Take $EI = \text{Constant}$.



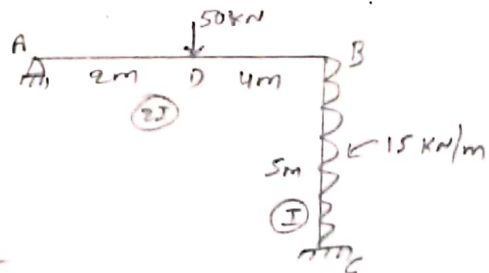
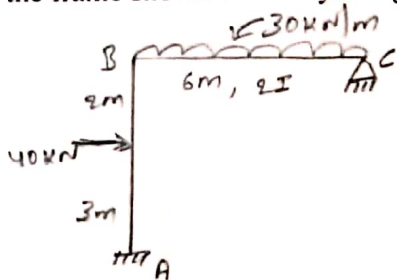
2. Analyze the truss shown below by using flexibility Method. Draw BMD. Take $EI = \text{Constant}$.



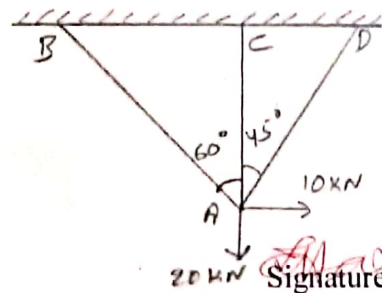
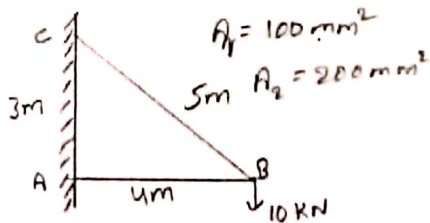
3. Analyze the continuous beam shown below by using Stiffness Method. Draw BMD. Take $EI = \text{Constant}$.



4. Analyze the frame shown below by using Stiffness Method. Draw BMD. Take $EI = \text{constant}$.



5. Analyze the truss shown below by using Stiffness Method. Draw BMD. Take $EI = \text{Constant}$.



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BGS Institute of Technology
Civil Engineering Department

Test: I

Semester: V

Section: A

Date: 08.09.2018

Subject Name & Code: ANALYSIS OF INDETERMINATE STRUCTURES (15CV52)

Instructions

Duration: 75minutes

Max. Marks: 30

i) Answer any TWO full questions selecting one from each part.

Question Number	Questions	Marks	CO	Levels
PART - A				
1	Analyze the frame shown in fig.1 by using Slope Deflection Method. Draw BMD. Take EI = Constant.	15	1	L ₅
OR				
2	Analyze the continuous beam shown in fig.2 by using Slope Deflection Method. Support B sinks by 10mm. Draw BMD. Take EI = 3000 kN/m ² .	15	1	L ₅
PART - B				
3	Analyze the frame shown in fig.3 by using Moment Distribution Method. Draw BMD. Take EI = Constant.	15	2	L ₅
OR				
4	Analyze the continuous beam shown in fig.4 by using Moment Distribution Method. Draw BMD. Take EI = constant.	15	2	L ₅

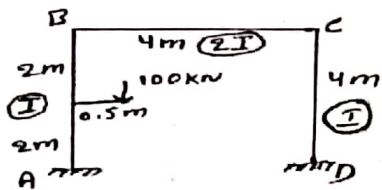


Fig.1

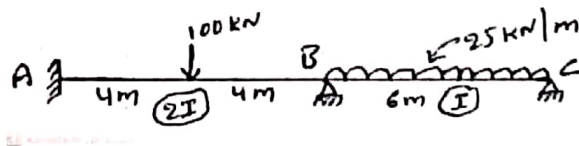


Fig.2

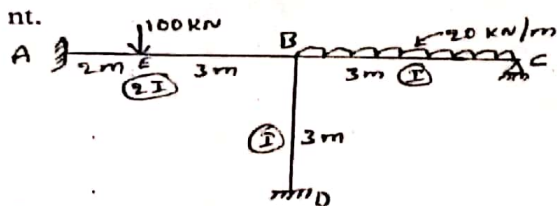


Fig. 3

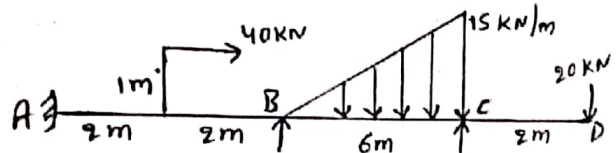



Fig. 4

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B G S Institute of Technology,
B G Nagara - 571448

Question Number	Solution	Marks Allocated
2.	<p><u>Fixed end moments</u></p> $M_{FAB} = -100 \text{ kN-m} \quad M_{FBA} = 100 \text{ kN-m}$ $M_{FBC} = -75 \text{ kN-m} \quad M_{FCB} = 75 \text{ kN-m}$ <p><u>Slope deflection equation</u></p> $M_{AB} = -105.625 + 15000 \theta_B$ $M_{BA} = 94.375 + 30000 \theta_B$ $M_{BC} = -70 + 20000 \theta_B + 10000 \theta_C$ $M_{CB} = 80 + 20000 \theta_C + 10000 \theta_B$ <p><u>Equilibrium equations</u> $M_{CB} = 0$</p> $80 + 20000 \theta_C + 10000 \theta_B = 0$ $M_{BA} + M_{BC} = 0$ $50000 \theta_B + 10000 \theta_C = -24.375$ $\theta_B = 0.0035$ $\theta_C = -0.042$ $M_{AB} = -100.375 \text{ kN-m}$ $M_{BA} = +105 \text{ kN-m}$ $M_{BC} = -105 \text{ kN-m}$ $M_{CB} = 0$ <p>BMD</p>	<p>04</p> <p>04</p> <p>02</p> <p>02</p> <p>02</p> <p>01</p> <p style="text-align: center;">15</p>


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 B G S Institute of Technology,
 B G Nagara - 571448

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Question Number	Solution	Marks Allocated																																																															
03.	<p><u>Fixed end moments</u></p> $M_{FAB} = -72 \text{ kN}\cdot\text{m} \quad M_{FBA} = 48 \text{ kN}\cdot\text{m}$ $M_{FCB} = 15 \text{ kN}\cdot\text{m} \quad M_{FBC} = -15 \text{ kN}\cdot\text{m}$ $M_{FBD} = M_{FDB} = 0.$ <p><u>Distribution factors</u></p> $(D.F)_{BA} = 0.41 \quad (D.F)_{BC} = 0.25$ $(D.F)_{BD} = 0.34$ <p><u>Moment distribution table</u></p> <table border="1" data-bbox="454 929 1204 1422"> <thead> <tr> <th>Member</th> <th>AB</th> <th>BA</th> <th>BD</th> <th>DB</th> <th>BC</th> <th>CB</th> </tr> </thead> <tbody> <tr> <td>D.F</td> <td>-</td> <td>0.41</td> <td>0.34</td> <td>-</td> <td>0.25</td> <td>-</td> </tr> <tr> <td>FEM</td> <td>-72</td> <td>48</td> <td>0</td> <td>0</td> <td>-15</td> <td>15</td> </tr> <tr> <td>Release @ C</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td>-15</td> </tr> <tr> <td>C.O</td> <td></td> <td></td> <td></td> <td></td> <td>-7.5</td> <td></td> </tr> <tr> <td>I.V</td> <td>-72</td> <td>48</td> <td>0</td> <td>0</td> <td>-22.5</td> <td>0</td> </tr> <tr> <td>Bal</td> <td></td> <td>-10.45</td> <td>-8.67</td> <td>0</td> <td>-6.375</td> <td></td> </tr> <tr> <td>C.O</td> <td>-5.225</td> <td></td> <td>0</td> <td>-4.335</td> <td></td> <td>0</td> </tr> <tr> <td>F.M</td> <td>-77.225</td> <td>37.55</td> <td>-8.67</td> <td>-4.335</td> <td>-28.87</td> <td>0</td> </tr> </tbody> </table> <p>BMD</p>	Member	AB	BA	BD	DB	BC	CB	D.F	-	0.41	0.34	-	0.25	-	FEM	-72	48	0	0	-15	15	Release @ C						-15	C.O					-7.5		I.V	-72	48	0	0	-22.5	0	Bal		-10.45	-8.67	0	-6.375		C.O	-5.225		0	-4.335		0	F.M	-77.225	37.55	-8.67	-4.335	-28.87	0	<p>04</p> <p>04</p> <p>06</p> <p>01</p>
Member	AB	BA	BD	DB	BC	CB																																																											
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C.O	-5.225		0	-4.335		0																																																											
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<p style="text-align: center;">  Head, Dept of Civil Engg. B G S Institute of Technology, B G Nagara - 571448 </p>		<p>15</p>																																																															

Question Number	Solution	Marks Allocated																																																																					
04	<p><u>Fixed end moments</u></p> $M_{FAB} = 10 \text{ kN}\cdot\text{m} \quad M_{FBA} = 10 \text{ kN}\cdot\text{m}$ $M_{FBC} = -18 \text{ kN}\cdot\text{m} \quad M_{FCB} = 27 \text{ kN}\cdot\text{m}$ $M_{FCD} = -40 \text{ kN}\cdot\text{m}$ <p><u>Distribution factors</u></p> $(D.F.)_{BA} = 0.67 \quad (D.F.)_{BC} = 0.33$ $(D.F.)_{CB} = 1 \quad (D.F.)_{CD} = 0$	04																																																																					
	<p><u>Moment distribution table</u></p> <table border="1" data-bbox="416 873 1209 1433"> <thead> <tr> <th>Joint</th> <th>A</th> <th colspan="2">B</th> <th>C</th> <th>D</th> </tr> <tr> <th>Member</th> <th>AB</th> <th>BA</th> <th>BC</th> <th>CB</th> <th>CD</th> <th>DC</th> </tr> </thead> <tbody> <tr> <td>D.F</td> <td>0</td> <td>0.67</td> <td>0.33</td> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <td>FEM</td> <td>10</td> <td>10</td> <td>-18</td> <td>27</td> <td>-40</td> <td>-</td> </tr> <tr> <td>Bal</td> <td></td> <td>5.36</td> <td>2.64</td> <td>13</td> <td>0</td> <td></td> </tr> <tr> <td>C.O</td> <td>2.68</td> <td></td> <td>6.5</td> <td>0</td> <td></td> <td></td> </tr> <tr> <td>Bal</td> <td></td> <td>-4.355</td> <td>-2.145</td> <td></td> <td></td> <td></td> </tr> <tr> <td>C.O</td> <td>-2.17</td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td>F.M</td> <td>10.5</td> <td>11</td> <td>-11</td> <td>40</td> <td>-40</td> <td>0</td> </tr> <tr> <td></td> <td>↺</td> <td>↺</td> <td>↻</td> <td>↺</td> <td>↻</td> <td></td> </tr> </tbody> </table>	Joint	A	B		C	D	Member	AB	BA	BC	CB	CD	DC	D.F	0	0.67	0.33	1	0	0	FEM	10	10	-18	27	-40	-	Bal		5.36	2.64	13	0		C.O	2.68		6.5	0			Bal		-4.355	-2.145				C.O	-2.17						F.M	10.5	11	-11	40	-40	0		↺	↺	↻	↺	↻		04
Joint	A	B		C	D																																																																		
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	<p>BMD</p> <p style="text-align: center;">  Head, Dept of Civil Engg, B G S Institute of Technology, B G Nagara - 571448 </p>	<p>06</p> <p>01</p> <p style="text-align: center;">(15)</p>																																																																					

USN:

BGS Institute of Technology
Civil Engineering Department

Test: II

Semester: V

Section: A

Date: 15.10.2018

Subject Name & Code: ANALYSIS OF INDETERMINATE STRUCTURES (15CV52)

Instructions

Duration: 75 minutes

Max. Marks: 30

i) Answer any TWO full questions selecting one from each part.

Question Number	Questions	Marks	CO	Levels
PART - A				
1	Analyze the continuous beam shown in fig.1 by using Kani's Method. Draw BMD. Take $EI = \text{Constant}$.	15	2	L ₅
OR				
2	Analyze the frame shown in fig.2 by using Moment Distribution Method. Draw BMD. $EI = \text{Constant}$.	15	2	L ₅
PART - B				
3	Analyze the frame shown in fig.3 by using Kani's Method. Draw BMD. Take advantage of symmetry.	15	2	L ₅
OR				
4	Analyze the continuous beam shown in fig.4 by using Flexibility Method. Draw BMD. Take $EI = \text{constant}$.	15	3	L ₅

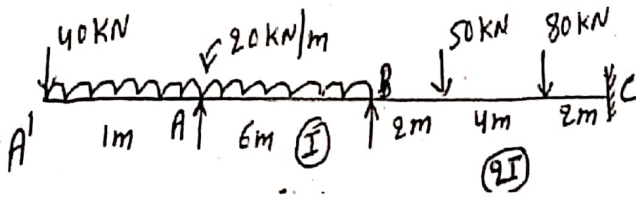


Fig.1

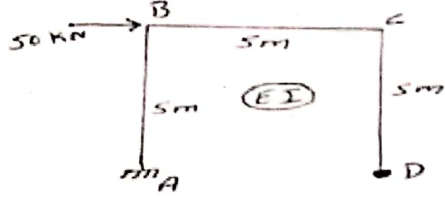


Fig.2

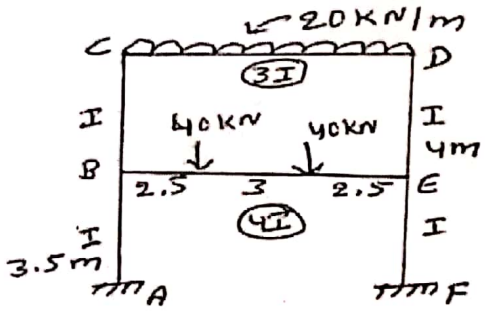


Fig.3

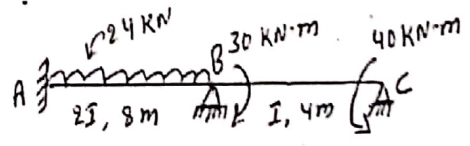


Fig. 4

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BGS Institute of Technology,
BGS Nagara - 571448

CBCS Scheme (VTU)

DEPARTMENT: CIVIL ENGINEERING

Scheme & Solution - TEST - II

Date: 15/10/2018

Semester: V Subject Title: Analysis of Indeterminate str. Subject Code: 15CV52

Question Number	Solution	Marks Allocated
1.	<p><u>Fixed end moments</u></p> $M_{FAB} = -60 \text{ kN}\cdot\text{m}$ $M_{FBA} = 60 \text{ kN}\cdot\text{m}$ $M_{FBC} = -86.25 \text{ kN}\cdot\text{m}$ $M_{FCB} = 108.75 \text{ kN}\cdot\text{m}$ $M_{AA'} = +50 \text{ kN}\cdot\text{m}$ <p><u>Rotation factors</u></p> $(U)_{BA} = -0.167$ $(U)_{BC} = -0.333$ <p><u>Rotation Moments</u></p> $M'_{BA} = 3.55 \text{ kN}\cdot\text{m}$ $M'_{BC} = 7.08 \text{ kN}\cdot\text{m}$ <p><u>Final moments</u></p> $M_{AA'} = 50 \text{ kN}\cdot\text{m}$ $M_{AB} = -50 \text{ kN}\cdot\text{m}$ $M_{BA} = 72.1 \text{ kN}\cdot\text{m}$ $M_{BC} = -72.1 \text{ kN}\cdot\text{m}$ $M_{CB} = 115.83 \text{ kN}\cdot\text{m}$ <p>BMD</p>	<p>4M</p> <p>1M</p> <p>2M</p> <p>3M</p> <p>2M</p> <p>2M</p> <p>1M</p> <p style="font-size: 2em;">15M</p>

Question Number	Solution	Marks Allocated
2.	<p><u>FEM</u> :- $M_{FAB} = M_{FBA} = M_{FBC} = M_{FCB} = 0$ $M_{FCD} = M_{FDC} = 0$</p> <p><u>Distribution factors</u> $(D.F)_{BA} = 0.5$ $(D.F)_{CB} = 0.57$ $(D.F)_{BC} = 0.5$ $(D.F)_{CD} = 0.43$</p> <p>Actual sway force = $S = 50 \text{ kN}$</p> <p>let, $M'_{CD} = -10 \text{ kN}\cdot\text{m}$ $M'_{BA} = -20 \text{ kN}\cdot\text{m}$</p> <p><u>Sway Moments</u> $M'_{AB} = -15.42 \text{ kN}\cdot\text{m}$ $M'_{BA} = -10.76 \text{ kN}\cdot\text{m}$ $M'_{BC} = 10.76 \text{ kN}\cdot\text{m}$ $M'_{CB} = 7.66 \text{ kN}\cdot\text{m}$ $M'_{FD} = -7.66 \text{ kN}\cdot\text{m}$ $M'_{DC} = 0$</p> <p>Cal. Sway force = $S' = -6.77 \text{ kN}$</p> <p><u>Final Moments</u> $M_{AB} = -113.8 \text{ kN}\cdot\text{m}$ $M_{BA} = -79.41 \text{ kN}\cdot\text{m}$ $M_{BC} = 79.41 \text{ kN}\cdot\text{m}$ $M_{CB} = 56.53 \text{ kN}\cdot\text{m}$ $M_{CD} = -56.53 \text{ kN}\cdot\text{m}$ $M_{DC} = 0$</p> <p>BMD</p> <p><i>[Signature]</i> Head, Dept of Civil Engg. J B G Institute of Technology, 3 B Nagar 567.1448</p>	<p>01M</p> <p>2M</p> <p>1M</p> <p>1M</p> <p>4M</p> <p>2M</p> <p>3M</p> <p>1M</p> <p>(15M)</p>

Question Number	Solution	Marks Allocated
3.	<p>FEM :- $M_{FCD} = -106.67 \text{ kN}\cdot\text{m}$ $M_{FBE} = -68.75 \text{ kN}\cdot\text{m}$ $M_{FDC} = 106.67 \text{ kN}\cdot\text{m}$ $M_{FEB} = 68.75 \text{ kN}\cdot\text{m}$</p> <p>Rotation factor $(U)_{BA} = -0.182$ $(U)_{CB} = -0.286$ $(U)_{BC} = -0.159$ $(U)_{CD} = -0.214$ $(U)_{BE} = -0.159$</p>	4M 3M
		5M
	<p>Final Moments</p> <p>$M_{AB} = 7.29 \text{ kN}\cdot\text{m}$ $M_{BA} = 14.58 \text{ kN}\cdot\text{m}$ $M_{BE} = -58.01 \text{ kN}\cdot\text{m}$ $M_{BC} = 41.42 \text{ kN}\cdot\text{m}$ $M_{CB} = 63.73 \text{ kN}\cdot\text{m}$ $M_{CD} = -63.73 \text{ kN}\cdot\text{m}$</p> <p>BMD</p>	2M 1M <u>15M</u>

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 B G S Institute of Technology,
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Question Number	Solution	Marks Allocated
4.	<p>$S.I = 2.$</p> <p>M_A and M_B are redundants</p> <p>$\delta L_1 = \frac{276}{EI}$ $\delta L_2 = \frac{309.3}{EI}$</p> <p><u>Flexibility coefficients</u></p> <p>$t_{11} = \frac{1.33}{EI}$ $t_{21} = \frac{0.67}{EI}$</p> <p>$t_{12} = \frac{0.67}{EI}$ $t_{22} = \frac{2.67}{EI}$</p> <p><u>Compatibility Equations</u></p> <p>$\delta L_1 + t_{11} R_1 + t_{21} R_2 = 0$</p> <p>$\delta L_2 + t_{12} R_1 + t_{22} R_2 = 0$</p> <p>$R_1 = M_A = -170.7 \text{ KN}\cdot\text{m}$</p> <p>$R_2 = M_B = -72.99 \text{ KN}\cdot\text{m}$</p> <p><u>Other reactions</u></p> <p>$V_A = 164.22 \text{ KN}$</p> <p>$V_B = 108.21 \text{ KN}$</p> <p>$V_C = -20.75 \text{ KN}$</p> <p>BMD</p>	<p>1M</p> <p>2M</p> <p>2M</p> <p>4M</p> <p>2M</p> <p>2M</p> <p>1M</p> <p>1M</p> <p><u>15M</u></p>


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 BGS Institute of Technology,
 B G Nagara - 571448

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BGS Institute of Technology
Civil Engineering Department

Test: III

Semester: V

Section: A

Date: 22.11.2018

Subject Name & Code: ANALYSIS OF INDETERMINATE STRUCTURES (15CV52)

Instructions

Duration: 75 minutes

Max. Marks: 30

i) Answer any TWO full questions selecting one from each part.

Question Number	Questions	Marks	CO	Levels
PART - A				
1	Analyze the frame shown in Fig.1 by using Flexibility Matrix Method with system approach. Draw BMD. Take EI = Constant.	15	3	L5
OR				
2	Analyze the rigid jointed frame shown in Fig. 2 by using Stiffness matrix method with system approach. Draw BMD. EI=Constant.	15	3	L5
PART - B				
3	Analyze the truss joint as shown in Fig. 3 by stiffness matrix method with system approach and also tabulate the member forces. C/s area of all the members are 1000 mm ² and E=2*10 ⁵ N/mm ² .	15	3	L5
OR				
4	Analyze the continuous beam shown in Fig.4 by using Stiffness Method. Draw BMD. Take EI = constant.	15	3	L5

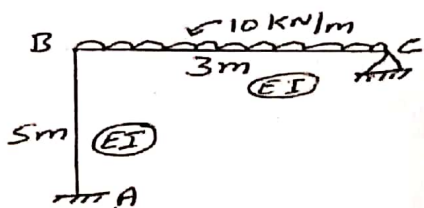


Fig.1

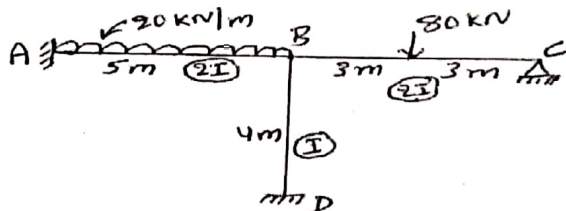


Fig.2

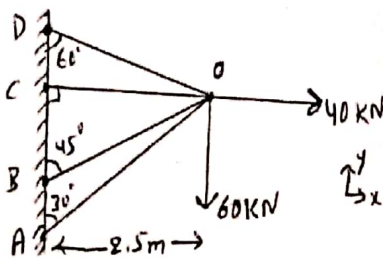


Fig.3

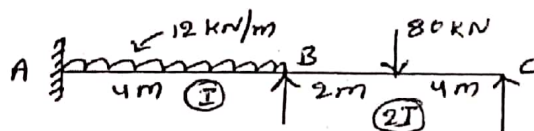


Fig.4

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Head, Dept of Civil Engg,
Signature of HOD
B G S Institute of Technology,
B G Nagar - 571442

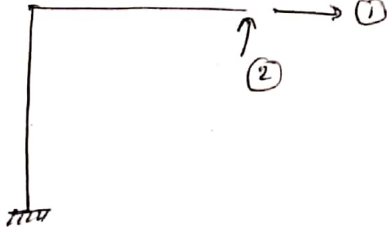

CBCS Scheme (VTU)

DEPARTMENT: CIVIL ENGINEERING

Scheme & Solution - TEST - III

Date: 22/11/2018

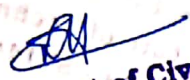
Semester: V Subject Title: Analysis of Indeterminate Str Subject Code: 15CV52

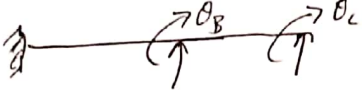

Question Number	Solution	Marks Allocated
1.	 $\Delta_1 = \frac{562.5}{EI} \quad \Delta_2 = \frac{-776.25}{EI}$ $[b] = \frac{1}{EI} \begin{bmatrix} 41.67 & -37.5 \\ -37.5 & 54 \end{bmatrix}$ $[P] = \begin{Bmatrix} -1.535 \\ 13.371 \end{Bmatrix}$ $M_{BC} = -4.91 \text{ kN}\cdot\text{m}$ $M_{BA} = +4.91 \text{ kN}\cdot\text{m}$ $M_{AB} = 2.71 \text{ kN}\cdot\text{m}$ <p style="text-align: center;">BMD</p>	<p>2m</p> <p>4m</p> <p>4m</p> <p>2m</p> <p>2m</p> <p>1m</p> <hr style="width: 50%; margin: auto;"/> <p>15m</p>
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Question Number	Solution	Marks Allocated
2.	$M_{FAB} = -41.67$ $M_{FBC} = -60$ $M_{FBA} = 41.67$ $M_{FCB} = +60$	2M
		1M
	$P_1 = -18.33$ $P_2 = 60$	1M
	$[K] = EI \begin{bmatrix} 3.93 & 0.67 \\ 0.67 & 1.33 \end{bmatrix}$	4M
	$\Delta_1 = \frac{13.417}{EI}$ $\Delta_2 = \frac{-51.74}{EI}$	2M
	$\theta_B = \frac{13.41}{EI}$ $\theta_C = \frac{-51.75}{EI}$	2M
	$M_{AB} = 30.9 \text{ kN}\cdot\text{m}$ $M_{BA} = 63.14 \text{ kN}\cdot\text{m}$ $M_{BC} = -76.6 \text{ kN}\cdot\text{m}$ $M_{CB} = 0$	
	$M_{BD} = 13.41 \text{ kN}\cdot\text{m}$ $M_{DB} = 6.71 \text{ kN}\cdot\text{m}$	2M
	BMD	1M
		1.5M

Question Number	Solution	Marks Allocated
3.	$P = \begin{Bmatrix} 40 \\ -60 \end{Bmatrix}$ $\Delta = \begin{Bmatrix} 0.000314 \\ -0.000859 \end{Bmatrix}$ $\theta_{oA} = 240^\circ \quad \theta_{oB} = 225^\circ$ $\theta_{oc} = 180^\circ \quad \theta_{oD} = 150^\circ$ $[K] = \begin{bmatrix} 170151.8 & 15602.8 \\ 15602.8 & 75549.6 \end{bmatrix}$ $P_{oA} = -23.48 \text{ (Comp)}$ $P_{oB} = -21.77 \text{ (Comp)}$ $P_{oc} = 25.12 \text{ (Tension)}$ $P_{oD} = 48.54 \text{ (Tension)}$	<p>1M</p> <p>4M</p> <p>6M</p> <p>4M</p> <hr/> <p>15M</p>

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Question Number	Solution	Marks Allocated
4.	$M_{FAB} = -16 \text{ kN}\cdot\text{m} \quad M_{FBC} = -71.11 \text{ kN}\cdot\text{m}$ $M_{FBA} = 16 \text{ kN}\cdot\text{m} \quad M_{FCB} = 35.58 \text{ kN}\cdot\text{m}$ <p>Choose θ_B and θ_C as redundants.</p> $\begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} = \begin{Bmatrix} -55.11 \\ 35.55 \end{Bmatrix}$ $[K] = \begin{bmatrix} 2.33 & 0.67 \\ 0.67 & 1.33 \end{bmatrix} EI$ $\theta_B = \frac{36.64}{EI} \quad \theta_C = \frac{-45.19}{EI}$  $M_{AB} = 2.36 \text{ kN}\cdot\text{m}$ $M_{BA} = 52.46 \text{ kN}\cdot\text{m}$ $M_{BC} = -52.46 \text{ kN}\cdot\text{m}$ $M_{CB} = 0$ <p>BMD</p>	<p>2m</p> <p>1m</p> <p>4m</p> <p>2m</p> <p>1m</p> <p>4m</p> <p>1m</p> <hr/> <p>15m</p>
<p style="text-align: center;">  Head, Dept of Civil Engg. B G S Institute of Technology, B G Nagara - 571448 </p> <p style="text-align: right;"><i>efee</i></p>		

Model Question Paper (CBCS Scheme)

Fifth Semester B.E. Degree Examination (CIVIL) Analysis of Indeterminate Structures (15CV52)

Time: 3 Hours

Max. Marks: 80

Note: Answer FIVE full questions, choosing one full question from each Module.

Module -1

1. A horizontal beam ABCD is loaded as shown in Fig. Q1. Plot SFD and BMD. Use slope deflection method. Support B settles by 10mm. $E = 2 \times 10^5 \text{ N/mm}^2$ $I = 2.4 \times 10^6 \text{ mm}^4$.

(16 marks)

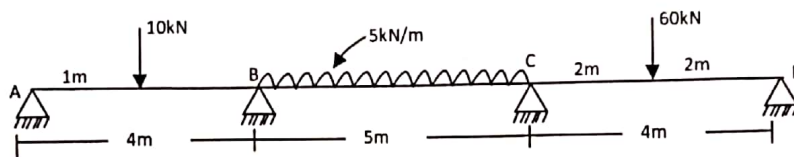


Fig. Q.1

OR

2. Analyze the frame shown in Fig. Q2 using slope deflection method. Draw BMD.

(16 marks)

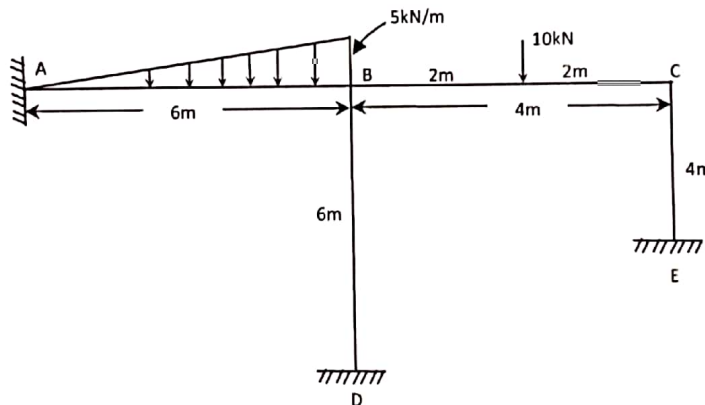


Fig. Q.2

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Module -2

3. Analyze the portal frame shown in Fig. Q3 using moment distribution method. Draw BMD

(16 marks)

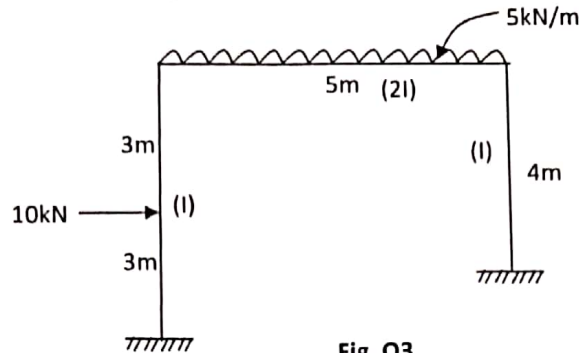


Fig. Q3

OR

4. Analyze the continuous beam shown in Fig. Q4 using moment distribution method. Draw SFD and BMD.

(16 marks)

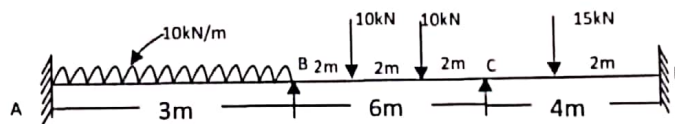


Fig. Q4

Module -3

5. Analyze the frame shown in Fig. Q5 using Kani's method taking advantage of symmetry. Draw BMD

(16 marks)

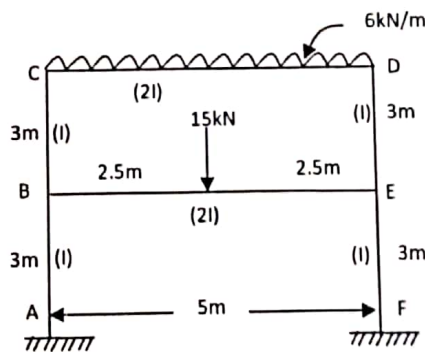


Fig. Q5

OR

6. Analyze the beam shown in Fig. Q6 using Kani's method. Draw BMD and elastic curve.

(16 marks)

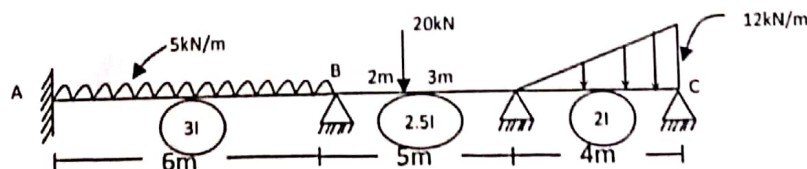



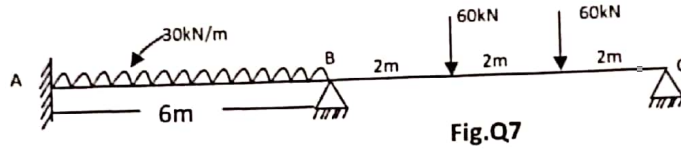
Fig. Q6

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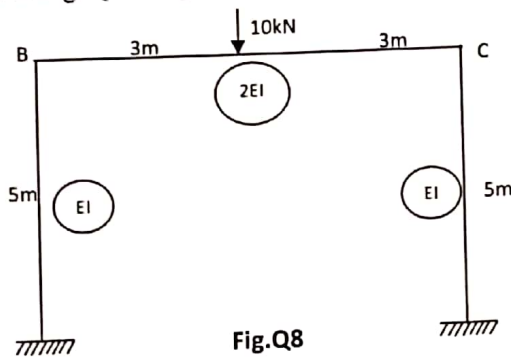
Module -4

7. Using flexibility matrix method, analyze the beam shown in Fig. Q7. Sketch SFD and BMD (16 marks)



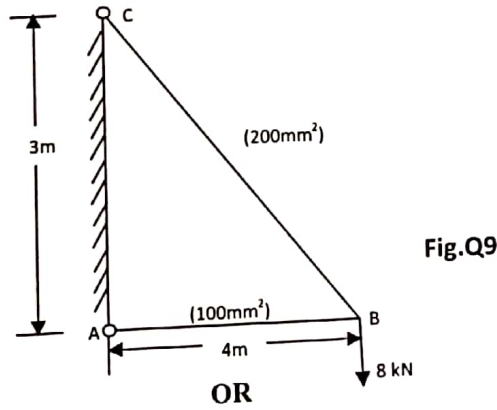
OR

8. Analyze the frame shown in Fig. Q8 using matrix flexibility method. Draw BMD (16 marks)



Module -5

9. Using stiffness method, determine forces in the members AB and BC of a pin jointed frame given in Fig. Q9. The cross sections are indicated in the brackets against each member. $E = 2 \times 10^5 \text{ N/mm}^2$ (16 marks)

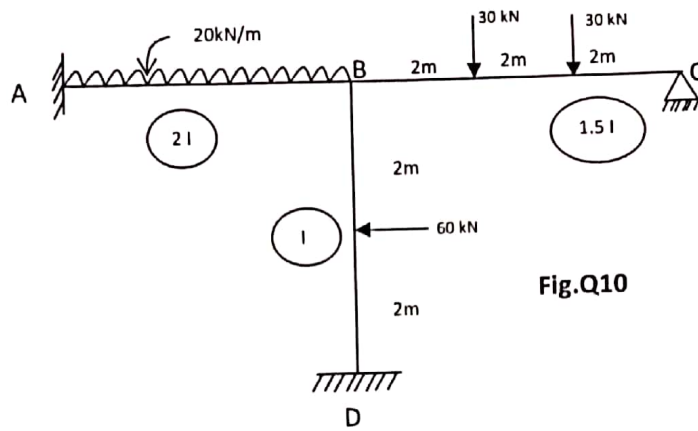


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10. Analyze the frame shown in Fig. Q10 using stiffness method. Draw BMD

(16 marks)



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CBCS Scheme

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Fifth Semester B.E. Degree Examination, Dec.2017/Jan.2018 Analysis of Indeterminate Structures

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing one full question from each module.

Module-1

- 1 Analyze the continuous beam shown in Fig.Q1 by slope deflection method. Draw BMD and EC.

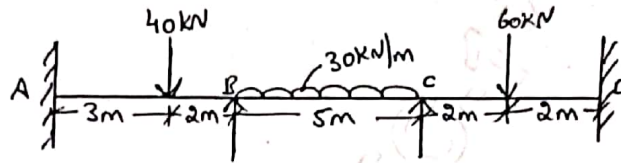


Fig.Q1 (16 Marks)

OR

- 2 Analyze the portal frame shown in Fig.Q2 by slope deflection method. Draw BMD.

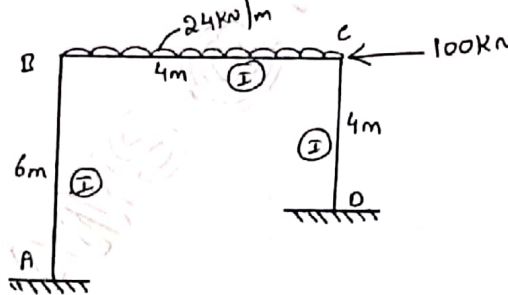


Fig.Q2 (16 Marks)

Module-2

- 3 Analyze the continuous beam by moment distribution method shown in Fig.Q3. The support 'B' sinks by 10 mm. Take $EI = 4000 \text{ kN-m}^2$. Draw BMD and EC.

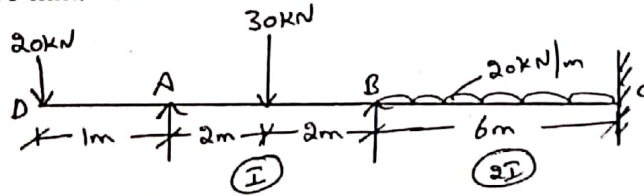


Fig.Q3 (16 Marks)

OR

- 4 Analyze the frame shown in Fig.Q4 by moment distribution method. Draw BMD.

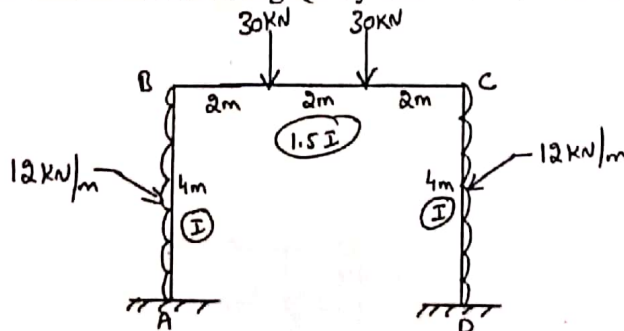


Fig.Q4 (16 Marks)

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Important Note : 1. On completing your answers, carefully draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

Module-3

- 5 Analyze the continuous beam by Kani's method. Shown in Fig.Q5. Draw BMD.

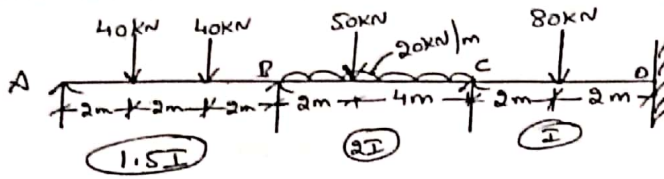


Fig.Q5

(16 Marks)

OR

- 6 Analyze the frame shown in Fig.Q6 by Kani's method. Draw BMD.

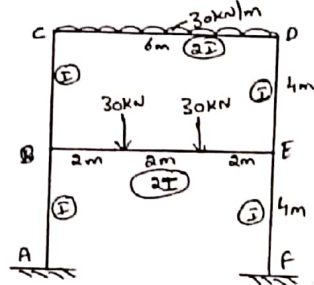


Fig.Q6

(16 Marks)

Module-4

- 7 Analyze the beam shown by flexibility matrix method. Draw BMD.

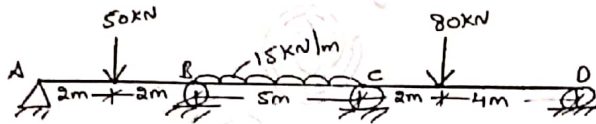


Fig.Q7

(16 Marks)

OR

- 8 Analyze the beam shown in Fig.Q8 by flexibility matrix method. Draw BMD.

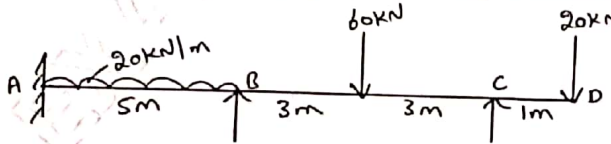


Fig.Q8

(16 Marks)

Module-5

- 9 Analyze the continuous beam shown in Fig.Q9 by stiffness matrix method. Draw BMD.

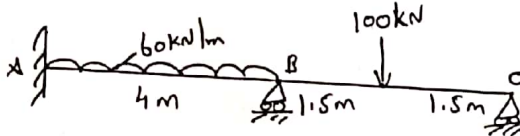


Fig.Q9

(16 Marks)

OR

- 10 Analyze the portal frame shown in Fig.Q10 by stiffness matrix method. Draw BMD.

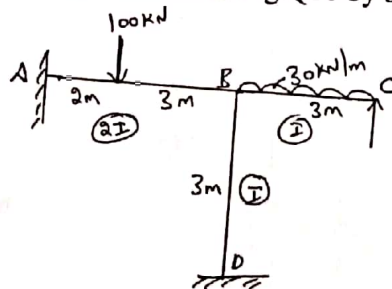


Fig.Q10

(16 Marks)

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CBCS Scheme

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Fifth Semester B.E. Degree Examination, June/July 2018 Analysis of Indeterminate Structures

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing one full question from each module.

Module-1

- 1 Analyze the continuous beam as shown in Fig.Q1 by slope deflection method and also determine its bending moment diagram and shear force diagram.

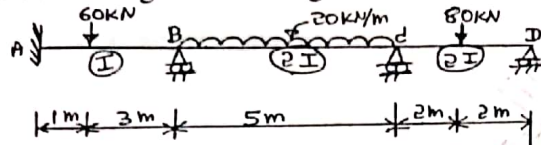


Fig.Q1

(16 Marks)

OR

- 2 Analyze the rigid jointed frame as shown in Fig.Q2 by slope deflection method and also determine its bending moment diagram.

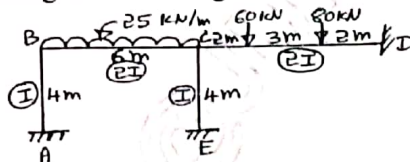


Fig.Q2

(16 Marks)

Module-2

- 3 Analyze the continuous beam as shown in Fig.Q3 by moment distribution method and also determine its bending moment diagram and shear force diagram.

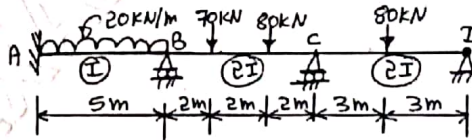


Fig.Q3

(16 Marks)

OR

- 4 Analyze the portal frame as shown in Fig.Q4 by moment distribution method and also determine its bending moment diagram.

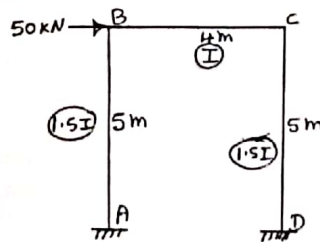


Fig.Q4

(16 Marks)

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Module-3

- 5 Analyze the continuous beam as shown in Fig.Q5 by Kani's method and also determine its bending moment diagram and shear force diagram.

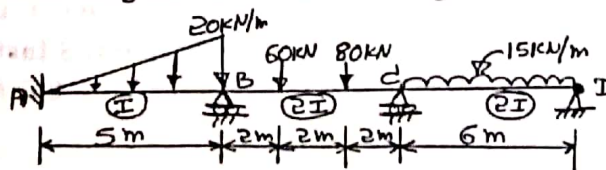


Fig.Q5

(16 Marks)

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Important Note : 1. On completing your answers, carefully draw diagonal cross lines on the remaining blank space.
2. Any revealing of identification, appeal to evaluator and/or equations written eg. 42+8 = 50, will be treated as malpractice.

OR

- 6 Analyze the portal frame as shown in Fig.Q6 by Kani's method by taking the advantage of symmetry and also determine its bending moment diagram.

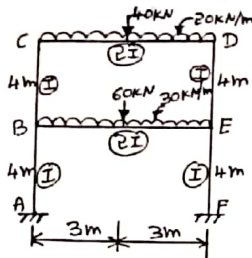


Fig.Q6

(16 Marks)

Module-4

- 7 Analyze the continuous beam as shown in Fig.Q7 by flexibility matrix method with system approach and also determine its bending moment diagram.

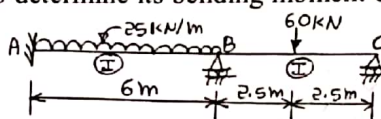


Fig.Q7

(16 Marks)

OR

- 8 Analyze the mill bent as shown in Fig.Q8 by flexibility matrix method with system approach and also determine its bending moment diagram.

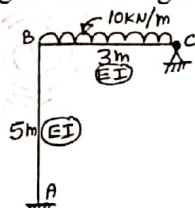


Fig.Q8

(16 Marks)

Module-5

- 9 Analyze the rigid jointed frame as shown in Fig.Q9 by stiffness matrix method with system approach and also determine its bending moment diagram.

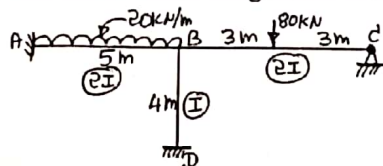


Fig.Q9

(16 Marks)

OR

- 10 Analyze the truss joint as shown in Fig.Q10 by stiffness matrix method with system approach and also tabulate the member forces. Cross section area of all members are 1000 mm^2 and $E = 2 \times 10^5 \text{ N/mm}^2$.

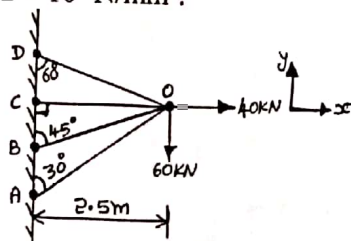


Fig.Q10

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(16 Marks)

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15CV52

Fifth Semester B.E. Degree Examination, Dec.2018/Jan.2019 Analysis of Indeterminate Structures

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 Analyze the continuous beam shown in Fig.Q.1 by slope deflection method and draw BMD. (16 Marks)

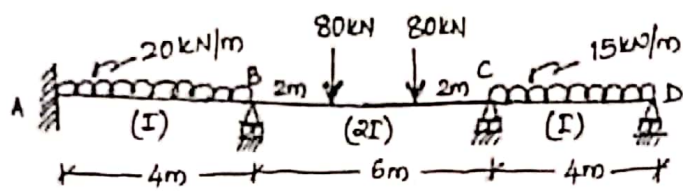


Fig.Q.1

OR

- 2 Analyze the rigid frame shown in Fig.Q.2 by slope deflection method and draw BMD. (16 Marks)

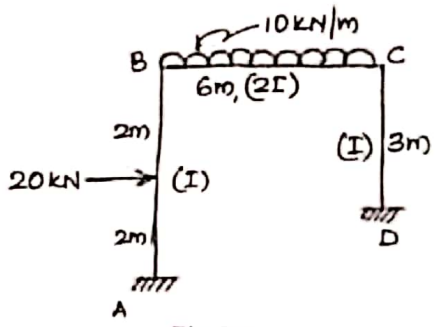


Fig.Q.2

Module-2

- 3 Analyze and draw BMD for the continuous beam shown in Fig.Q.3 by moment distribution method if support 'B' sinks by 30mm and support 'C' sinks by 20mm. Take $EI = 24,000 \text{ kNm}^2$. (16 Marks)

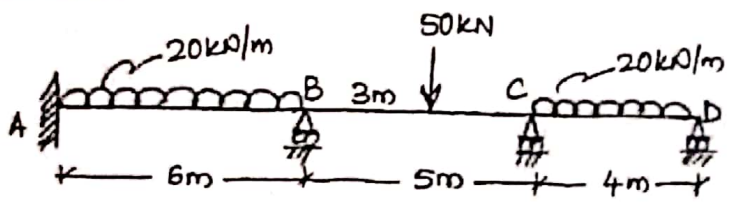


Fig.Q.3

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Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice

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OR

- 4 Analyze the rigid frame shown in Fig.Q.4 by moment distribution method and draw BMD. (16 Marks)

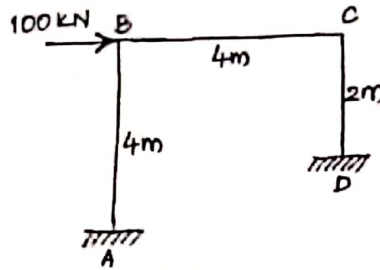


Fig.Q.4

Module-3

- 5 Analyze and draw BMD for the continuous beam shown in Fig.Q.5 by Kani's method, if support 'B' sinks by 10mm and $E = 2 \times 10^5 \text{ N/mm}^2$, $I = 1.2 \times 10^4 \text{ m}^4$. (16 Marks)

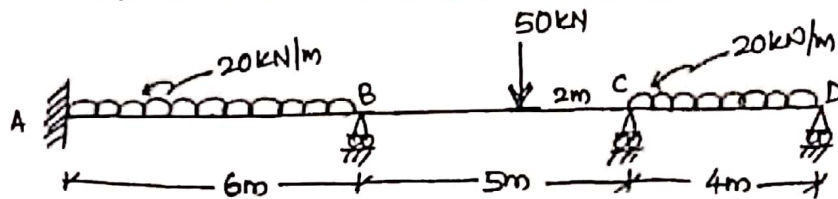


Fig.Q.5

OR

- 6 Analyze the rigid frame shown in Fig.Q.6 by Kani's method and draw BMD. (16 Marks)

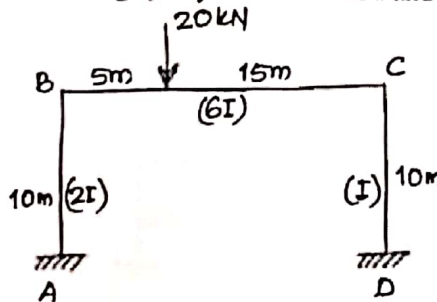


Fig.Q.6

Module-4

- 7 Analyze the continuous beam shown in Fig.Q.7 by matrix flexibility method using system approach and draw BMD. Take moments as redundants. (16 Marks)

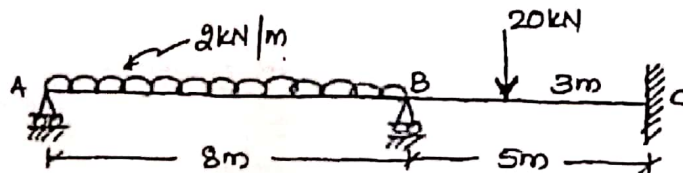


Fig.Q.7

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OR

- 8 Analyze the pin-jointed truss shown in Fig.Q.8 by matrix flexibility method of system approach and determine forces in all the members. Take force in member 'OA' as redundant. (16 Marks)

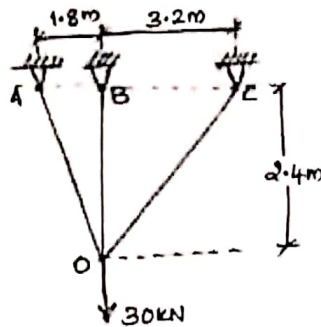


Fig.Q.8

Module-5

- 9 Analyze the rigid frame shown in Fig.Q.9 by matrix stiffness method and draw BMD. (16 Marks)

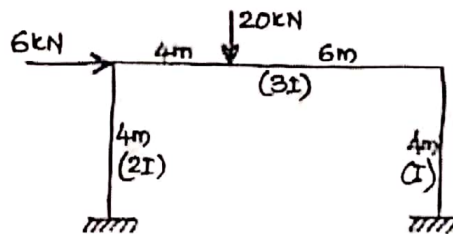


Fig.Q.9

OR

- 10 Analyze the pinjointed frame shown in Fig.Q.10 by matrix stiffness method and find forces in all the members. The numbers in parentheses are the C/S areas of members in sqmm. (Take E = constant). (16 Marks)

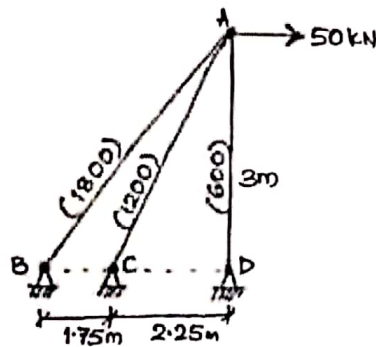


Fig.Q.10

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